# Approaching Freedom:

## unified paradigms of choice in psychology, physics and technology

by Robert Kovsky

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Introduction: This approach to freedom is guided by psychological investigations of Jean Piaget.

Swiss psychologist Jean Piaget (1896-1980) and his collaborators investigated the genesis and development of intelligence in children during growth from infancy to adolescence. Using an approach of progressive construction, their researches focused on both general and particular mental functions. Important publications include *The Origins of Intelligence in Children*; *The Child's Construction of Reality*; *Play, Dreams and Imitation in Childhood* (*La Formation du Symbole)*; *The Child's Conception of Movement and Speed*; *The Child's Conception of Number* and *The Psychology of the Child.*

A Piagetian construction is based on observations of children's activities involving movements, material objects, puzzles and problems; and investigations include questioning that encourages a child's self-examination. Such a construction starts with rudimentary or anticipatory features of the function of intelligence (e.g., functions of space or number) that is observed at the youngest age and then adds additional features in a step-by-step progression that tracks the children's growth. Thus, the construction of the child's conception of number starts with perceptual constancies (identifying images that are "the same") and progresses to repetitions (series), counting (ordinal numbers) and classification (cardinal numbers). The features are combined in functions of intelligence, which change during the construction, as reflected in the child's development. Many functions of intelligence and their development in children are organized by such methods.

The present approach to freedom does not track development in children. In contrast to Piaget's constructions of speed, number and reality, the aspect of intelligence that is investigated here — freedom — has been shrouded in such confusion that its character has been misrepresented as "free will" and some have even denied it altogether Notwithstanding difficulties and differences, this investigation does emulate those of Piaget in its combination of observational, introspective and inventive methods; in its use of rudimentary and anticipatory forms and paradigms that are directed towards development of more complete and complex combinations; and in a step-by-step and accumulative approach.

A. Psychological choices are organized by a critical selection process during which multiple possible movements change into a single actual movement.

1. Examples of choices in actual life (food market, ping pong, Judge)

Psychological principles of choice or selection are illustrated by three examples, picking a snack at a market, a ping pong player and a federal court judge.

(a) Suppose that I am in a food store intending to buy a snack and standing in front of shelves or bins containing packages of candy, nuts, chips and similar items. At a certain moment, my arm will extend in a specific direction and my hand will pick one package that I will then carry to the checkout counter.

(b) Suppose that a ping pong player has just completed a stroke, having hit the ball during a volley in a tournament game. He is seeing his opponent in motion while performing the return stroke and, in anticipation, he is starting to move his lower body to the left (in response to the opponent's motion) while, with his right arm holding the paddle, he starts into a swing that, as it continues, he will turn into one of several ping pong strokes, such as a loop, a drive or a smash.

(c) Suppose that a United States District Court Judge is reviewing a request or Motion made by one side in a civil lawsuit, asking the Judge to order that a certain procedure be carried out earlier in the case than is the norm set forth in the rules, which also allow for exceptions "for good cause." The other side opposes the Motion. The Judge can grant the Motion or deny the Motion; and she can find sufficient justification in the facts and in legal principles to support either decision.

In each example, two or more possible courses of action are changing into a single actual course of action. When one of the possible courses of action becomes the actual course of action, the other possibilities cease to exist even as a possibility, at least for this selection. Each change is produced by a ***person exercising freedom***.

My choice of a snack is an example of a general kind of freedom that people exercise in all markets. The ping pong player's stroke is an example of a general kind of freedom that people exercise in all games and sports. The Judge's ruling is an example of a general kind of freedom that authorized officials exercise in all courts and during many governmental determinations. I suggest that such exercises of freedom make up a substantial part of many a person's conscious activity. I suggest that markets, game rooms, sport fields and courts of law are maintained to organize and cultivate such exercises of freedom.

2. Common features of choices

The three examples manifest common features that are combined in a mental model or paradigm. I suggest that the paradigm applies to many actual choices and can be developed for further applications.

a. **Selection**. A ***selection*** is a ***process*** during which two or more possible courses of action turn into a single actual course of action. A process is a succession of movements and conditions that occurs in a compact period of time. The phrase "turn into" denotes a transformation of movements. The simplest course of action is a single movement, such as picking a snack from a shelf or a bin in a market. Prior to the selection, there are two or more possible picking movements, perhaps chips, nuts or chocolate. After the selection, one of those possibilities has become ***the actual movement***. The other possibilities have ceased to be possible, at least during this selection; they may return another day.

b. **Critical moment**. A selection occurs during a period of time, the ***critical moment***. The critical moment is difficult to define directly so an indirect definition is used. There was a time before the selection when there were possible deeds but no actual deed. At a later time after the selection is complete, e.g., in front of the cashier, possibilities no longer exist but an actual movement has been performed. After completion of the selection, there has occurred, historically, a succession of stages in time and the "before" stage is separated from and distinct from the "after" stage. In between the "before" and "after" stages, there is an interval of time during which the selection occurs. It is during this interval that the critical moment occurs. The critical moment may last a split second (in the case of the ping pong player), a few seconds (in the market) or may extend over a prolonged period of deliberation (that of the Judge). A transformation occurs within an interval of time that begins with one state of affairs, in which there are two or more possibilities, and which ends with a different state of affairs, in which there are no possibilities but one actuality.

Sometimes edges of a critical moment are well-defined. A process of selection may start at a particular instant, e.g., upon opening an envelope containing an offer. A ***deadline*** identifies a time by which a selection must be completed. Deadlines can be rigid or elastic. The ping pong player has a rigid deadline set by the arriving ball. I can't spend all day picking a snack and the Judge must keep current with a heavy caseload, but these deadlines are more elastic; a short delay creates no problem as it would with the ping pong player.

Regardless of definition, a transformation occurs in a brief time interval. We encounter difficulties when we try to look into such a critical moment. There seems to be an unbridgeable gap between (1) a person having multiple possible courses of action and (2) the person carrying out a single selected course of action through bodily movement.

Some have discussed this gap as part of the "mind-body problem." The gap identifies a problem we have in understanding how possible deeds turn into actual deeds. Factually, every day, in many ways, every person is continually bridging that gap. Physical models of selection discussed below suggest a technological means for investigating and bridging the gap.

c. **Influences**. The actual result of a selection often depends on a person's consciousness of matters that the person can identify, e.g., in later conversation or in a written description. A person may (but need not) consider such matters during a selection; and the person's considerations typically lead to the actual result. As a spotlight on the way this works: sometimes a person neglects to consider a certain matter during a decision and recognizes later that, if that matter had been considered, the actual result would have been different — and better. Without attempting an exact definition, I say that such matters that are consciously considered are ***influences*** on the selection.

Using the examples, while selecting a snack, I may be influenced, e.g., by habit or a desire for novelty, by taste preferences, remarks of friends or advertising, by a reduced price or by theories of nutrition and health. Perhaps I buy peppermint-flavored disks made of "pure organic cane sugar" to suck on while I drive my car.

The ping pong player selecting a stroke may be influenced by the difficulty of returning the opponent's shot, by the opponent's position across the table, by the player's recent history of success or failure of various strokes, by the score of the game, or by a sense of prowess in comparison with that of the opponent. Perhaps the ping pong player goes for the loop because he is behind 9-8 (11 points wins the game), he feels that he can't make a mistake, he sees that he can meet the incoming ball squarely with a loop and the loop is his most reliable stroke.

The Judge deciding the Motion asking for an early use of a procedure may be influenced by Opinions (formal explanations of decisions) written by other judges deciding a similar issue in earlier cases, by factual or legal presentations of the lawyers, by expectations about the future course of events in the case, or by general judicial policies such as a policy of discouraging requests for special treatment. Perhaps the Judge denies the Motion without giving a reason but actually because the lawyer for the party making the Motion is trying to overwhelm the other side; and the Judge also schedules a case management conference (a private meeting with the lawyers) during which she will deal with the issue presented by the Motion in her own way.

d. **Repetitive cycling**. An additional feature of repetitive cycling can organize ongoing activity. Each cycle, a similar selection is made. In certain situations, it is possible to vary influences and investigate how varying influences lead to different selections. The clearest examples involve ***approximately repetitive selections***. That is, a particular situation involves a certain selection that occurs repeatedly and that has the same possible results but where influences vary. With an influence that is undergoing a progressive variation, the actual result remains the same, up to a certain point; and then, suddenly, a further variation in the influence leads to a different actual result. Such activity is modeled by repetitive cycles and ***cyclical selections***.

These principles are illustrated by adapting the three examples to illustrate approximately repetitive selections and including also, in each example, a second kind of selection. Suppose that I am at a different place in the market and selecting potatoes from a heap of potatoes, examining one potato after another and picking out the best; but if none meet my standards, I will leave the potato bin and think of something else to cook. Suppose the ping pong player is now practicing with a partner who repetitively drives balls into a certain area of the table for the player to try a variety of responsive strokes; but if nothing is challenging him, the ping pong player will try a different exercise. Or suppose the Judge is now quickly deciding Motions; but if one case appears unusual, the Judge may schedule a hearing.

In each of the adapted cases, selections occur repetitively (cyclically) and under a set of circumstances that is largely fixed. Yet, each selection is unique. The repetition and similarity enable the person to aggregate and to compare and contrast a large number of examples, seen together as a single subject matter. The aggregation is what influences the second kind of selection that results, e.g., in my leaving the potato bin, in the ping pong player trying a different exercise and in the scheduling of a hearing by the Judge.

With repetitive cycling, it is possible to introduce ***comparing*** and ***contrasting*** individual choices. These foundational operations generate ***resemblances***. When comparisons between two movements, objects or images appear to be exact and no contrasting features are discerned, the two movements are said to be ***identical*** or "the same." Close resemblances occur when the two appear to be identical except for certain specific distinguishing features. Movements may have identical endpoints but manifest different courses of movement to get there. Other resemblances may be more remote, such as analogies, metaphors and recollections.

## 3. A deeper investigation into ping pong

Of the three examples of choices, ping pong is the only one that involves ***momentum***, especially the momentum of the ping pong ball that is the focus of movements of the players. A deeper investigation into ping pong illustrates the problems of freedom that require a new approach.

The focus of investigation is the ping pong ***stroke***, which occurs each time a player attempts to strike a ball in play. In an empirical or "natural science" approach, a ping pong game is constituted by (made up of) a ***sequence of strokes***, which could be numbered. The definition of "in play" includes services, the stroke that starts a point. In this discussion, I focus on volley strokes that occur after service.

During a volley stroke, a player moves purposefully (to win!) and spontaneously (here comes the ball!). A spontaneous action is one decided at the moment of action, rather than by pre-arrangement. I describe such an exercise of freedom as "purposeful, spontaneous action." By exercising freedom, players ***construct or produce a game*** of ping pong, stroke by stroke, point by point.

A stroke is centered around the paddle-impact, identified by the momentary, audible contact ("click") between the paddle and the ball. Each individual stroke is separate and distinguishable from every other stroke. The strokes are collected into a compact, well-defined class of strokes. In ping pong, freedom comes conveniently packaged for investigation.

For purposes here, facts about ping pong strokes fall into three classes: (1) objects and perceptions; (2) muscular movements and (3) purpose and intentions.

The first class of stroke-facts collects ***objects and perceptions*** involved in the ping pong strokes. Here, the word "object" is used broadly and applies to physical material objects; to individual persons and people in general; to words, text and meanings; to natural laws, juridical laws (enforced by courts of law) and geopolitical states. Objects organize perceptions that are based in the senses.

A major source for information about objects in ping pong is *The Laws of Table Tennis* found at the website of the International Table Tennis Federation, http://www.ittf.com. The Laws define terminology; describe the equipment; require, authorize and/or prohibit certain acts during play; and prescribe procedures for play. "Everyone has to agree" to The Laws of Table Tennis to play. That's what makes ping pong possible. Of course, laws can be relaxed or changed by private agreement but The Laws are necessary for any such modifications.

The second class of stroke-facts collects ***muscular acts*** involved in possible ping pong strokes. It is a fact of ping pong that nearly all strokes made by a player with substantial experience can be classified according to a particular ***stroke style*** such as the drive, the block, the flip, the loop, the chop, the push and the smash. See http://tabletennis.about.com/library/glossary/bl-glossary.htm for descriptions of these stroke styles. For example, in a loop, the player quickly raises the paddle to meet the ball in flight, "just skimming the ball on the way up," and thus seeks to return the ball to the adversary with hopefully-surprising spin; there is little attempt to speed up the ball by forceful impact and the loop is "more subtle" than a drive (direct shot, forceful but controlled, e.g., to stay low near the table) or a smash (more powerful than a drive and with control sacrificed for maximum speed).

A person's muscular acts involved in a particular activity are organized by the concept of ***repertoire***. For example, it is possible to state a repertoire of stroke styles for ping pong, R = {drive, block, flip, loop, chop, push, smash}. The repertoire is open for additional stroke styles and may include practical receptacles, e.g., "miscellaneous" or "freaky."

The third class of stroke-facts collects ***purposes and intentions*** involved in ping pong strokes. A ***sustained purpose*** is presumed to be constant throughout the activity and there are also ***transient particular intentions***. I borrow a legal definition and say that an intention is a mental determination to realize a purpose through particular means and particular acts.

Each player is always motivated by a sustained purpose ***to win the game***. The players have ***competing*** purposes to win and the competition is a central "hook" on which everything hangs: take away the competition and there is nothing of substance left to a game of ping pong. There is, therefore, the always-overarching goal of winning the game, what I call a sustained purpose.

A player's sustained purpose to win the game is present during every stroke. In addition, there are transient intentions that are similar in feeling to the sustained purpose but that last only a short time. For example, a player may have the transient intention that the then-current stroke send the ball to impact the opponent's side of the table close to an outside edge near the net. After that stroke has ended and a new stroke is being prepared, the former transient intention disappears and a new transient intention forms. A transient intention need not be clear and some transient intentions cannot be expressed in words, feeling more like a need to focus action, e.g. (if a desperate intention were to be expressed in words) "gotta get the paddle on this..."

The foregoing analysis defines three classes of stroke-facts: (1) objects, (2) acts and (3) intentions. The classes of facts show parallel structures; that is, the three classes have a structural form in common. In each structure, there is a general conceptual class along with particular members. Objects organize perceptions; repertoire organizes acts; purpose organizes intentions. Structures of objects can be developed in great detail, e.g., by keeping records of games. Structures of acts and intentions can also be developed, e.g., with a coach.

I suggest that, in my natural science view of ping pong strokes, a player is simultaneously accessing three streams of experiential material: objects, acts and intentions. During a stroke, the player selects from each of the three streams and combines the selections by producing the particular stroke, with its particular object-based perceptions, particular stroke style out of the repertoire and particular intention as an expression of sustained purpose.

The foregoing description divides the player's activity into "selects" and "combines" but this a shortcoming based on an inability to describe more precisely. "Selecting" and "combining" are going on at the same time.

After consideration of possibilities for further analysis, I conclude that, from a natural science viewpoint, ***a ping pong stroke is irreducible***. That is, I know of no way to state how a particular ping pong stroke is produced and there does not appear to be any way to divide the problem into parts. It can be described in some generic, clumsy way as "selecting" and "combining," but there is nothing in these words that particularizes selections and combinations for a particular stroke. The structure of objects and the repertoire of stroke styles suggest that some guidances can be stated ("if the ball is in a slow high bounce, go for the smash"), but there is no general means for decision. In other words, ***there is something going on to produce the ping pong stroke but it is hidden from view.***

There is an additional important feature: whatever is going on to produce the ping pong stroke is working with ***remarkable speed***. A player has only a few tenths of a second to produce a ping pong stroke, during which, presumably, the complex co-ordinations previously described are taking place.

To state the conclusion in a more detailed way: each of the three streams of objects, acts and intentions has its own structure, as indicated above. Material from the three streams is being combined with remarkable speed, but no way appears to organize the three structures together, fast or slow. I cannot conceive of how structures of objects–perceptions, repertoire–acts and purpose–intentions get organized together to produce a particular stroke so quickly. It would seem that the whole repertoire of stroke styles must needs be accessed before a particular stroke is selected and something similar with the structure of objects. It is more difficult to state how purpose and intentions figure in; but figuring them in is not likely to simplify the situation.

There are three streams (objects, acts, intentions) that "somehow" get combined. We can't see how but there it is: Chris put a lot of spin on the ball with a flip but Jerr anticipated the bounce and returned with a killer drive to the opposite corner.

I suggest that the production of the stroke is an exercise of freedom but that the central part of the stroke cannot be seen. The ping pong stroke presents the Question of Freedom in individualized, concrete actuality. I suggest that ***freedom is hiding inside the center of each ping pong stroke***.

Next, I apply suggestions of child psychologist Jean Piaget about ***concept formation*** in children to investigate the nature of the "hiding" of freedom inside ping pong strokes. Piaget developed a "constructivist" psychology, teaching that we construct our experiences and our reality and that the progressive course of construction can be discerned through observations of children.

To sum up the prior discussion: the production of a particular ping pong stroke requires a player to combine perceptions, acts and intentions; and, indeed, players accomplish such combinations and produce strokes; but we cannot conceptually follow the threads of combination or trace connections in any systematic way. We have a conception ***that*** a ping pong stroke is going on and that the combination is being produced; but we cannot form a conception of ***how*** it is going on.

Contrast our inability to conceive of how we produce a ping pong stroke with knowledge of how bicycles work. I presume that every intelligent adult knows "how" bicycles work. A person presses on a pedal with his or her foot; the force is passed through the pedal crank to the pedal gear, thence to the chain, thence to the rear wheel gear and finally through the rear wheel to push the tire against the pavement and propel bicycle and rider forward. There is a ***continuous passage of force*** from the person's foot – through parts of the mechanism – to the pavement (with gears adjusted to provide the best mechanical advantage); and the ***continuity*** is the key.

A person need not study texts to learn how bicycles work. People learn how bicycles work by riding them, looking at them and working on them. Many a person turns a bicycle upside down so that the wheels are uppermost and the frame rests on the seat and handles. While the bicycle is in such a position, hold the rear wheel with one hand and move a pedal crank with the other hand. Co-ordinate the motions of the hands. Watch the chain first stretch and then relax and see how the parts touch and move one another. Ah-ah.

In bicycles, there is a clear path from knowing ***that*** a bicycle works to knowing ***how*** a bicycle works. That step is taken by acquiring knowledge through direct experience. In the case of a bicycle that knowledge is based (1) on a concept, that of force, and (2) on a principle of continuity so that the force is passed from one part of the system to another and so forth. The basis of knowledge is activity of muscles and senses that enables an intelligent person to play with bicycles.

As stated by Piaget, at about age 5-7, a child has only limited skills in forming concepts and applying them to tasks of ordinary life. A child of that age mostly uses a style of concept formation called ***juxtaposition***. Details are not organized but simply "stuck together." Later, a growing child develops additional means of concept formation, called ***synthesis***, that culminates in the mental activity of an adult where details are organized into ***structures of relationships***. Explanations of synthetic relationships provide answers to children's "why" and "how" questions. "We can say that childish conceptions are the result of the juxtaposition and not of the synthesis of a certain number of elements which are still disparate and will only gradually come into relation." Piaget, *Judgment and Reasoning in the Child* (orig. pub. 1924, English reprint 1968) at 157.

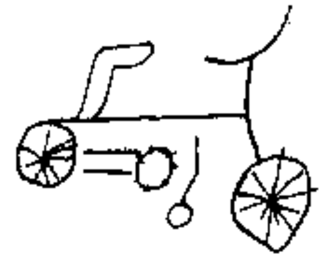
Adapting Piaget's analysis,. I set up "juxtaposition" and "synthesis" as ***two kinds of concept formation***. First there is concept formation through juxtaposition, where features are experienced as being simultaneously present but without more. Through additional concept formation, that of synthesis, a concept formed by juxtaposition is infused with relations and becomes embedded in a structure of relationships with other concepts.

Juxtaposition constructs ***facts*** in an infant's intelligence. "For any two phenomena perceived at the same moment become caught up in a schema which the mind will not allow to become disassociated, and which will be appealed to whenever a problem arises in connexion with either of these two phenomena." *Judgment and Reasoning* at 229. [I interpret Piaget's somewhat variable term ***schema*** to include both spatial structures and also temporal structures of movements, e.g., melodies discussed in the Zuckerkandl excerpts below.]

"Juxtaposition" is one aspect of this style of concept formation; another is "syncretism." "[S]yncretism and juxtaposition constitute two phases alternating over indefinite periods in the mind of the child." (*Id*., at 59)

"[S]yncretism is a vision of the whole which creates a vague but all-inclusive schema, supplanting the details." (*Id*.) Syncretism, "which makes the child connect everything with everything else, and prevents him from making the excisions and distinctions necessary to analytic thought, will have the natural consequence of making him concentrate heterogeneous elements within a single word." (*Id*., at 240). For example, a "child, unable to choose between two contradictory explanations of one and the same phenomenon, agrees to both simultaneously and even fuses them into each other." (*Id*. at 242.)

The adjacent image of a bicycle drawing appears in Piaget, *The Child's Conception of Physical Reality* (1927) and Gruber & Vonèche's extracts therefrom. It is similar to the drawing discussed below. Such childish drawings illustrate limitations of the juxtaposition style of concept formation.



"The drawing of a bicycle by a child of 6, for example, will show, in addition to the frame and the two wheels, the pedals, a chain, a cog-wheel, a gear. But these details are juxtaposed without any order; the chain is drawn alongside of the cog-wheel instead of being correctly inserted, and the pedals are suspended in mid-air instead of being fixed. Thus everything happens as though the child really felt the relations in question, knew that the chain, the pedals and the cogwheel were necessary to set the machine in motion and that these different pieces 'went together.' But this is as far as his consciousness of the relations goes; it does not extend to a precise knowledge of the details of the insertion and contact. The drawing is therefore comparable to the thought, and the thought to the drawing. Both juxtapose instead of synthesizing." *Judgment and Reasoning* at 58.

I suggest that, when investigating an exercise of freedom involved in a ping pong stroke, I am in a position similar to that occupied by a 6-year-old child. Each of us can form a concept through juxtaposing disparate elements; but relationships between the elements are "lacking in logical necessity." It is with me thinking about ping pong strokes as with the child drawing the bicycle who "really felt the relations in question ... that these different pieces 'went together,'" but who could not obtain "a precise knowledge of the details." In ping pong, the different pieces are objects, acts and intentions and there is something organized going on that I "really feel" but about which I cannot obtain "precise knowledge." I do not understand how object, act and intention make "insertion and contact" with one another. Lacking understanding, I have to "concentrate heterogeneous elements within a single word," namely, "freedom."

The psychological analysis clarifies the cloud around freedom hiding at the center of a ping pong stroke. Unlike the children, I have fully-developed capacities for synthesis and operational thought. They just don't work on exercises of freedom that produce ping pong strokes. Juxtaposition works well enough to form a syncretic concept and that is what I have.

[The original version of this section was published in January, 2005 as part of the Introduction to *Researches in Personal Freedom*. The present version incorporates more recent developments.]

B. Critical selection processes in paradigms of physics and mathematics have parallels with psychological choices, leading to the more highly activated Quad Net paradigm that embodies principles of freedom..

Overview. Four preparatory steps lead to quadnets.

Previous descriptions of psychological selection processes have been based on muscular movements, bodily feelings and other imagery of actual life. In this part, distinctly different domains are constructed that are based on physics models. Psychological descriptions of selection processes and physics models of selection processes have convergent features that lead to features of freedom. In physics models, such features arise from fundamental ***critical point*** properties of materials with a collective character, such as magnets and closed containers containing liquid water and steam.

Features of physics models suggest ways to overcome the problems of freedom previously discussed in the analysis of ping pong and in other psychological examples. In the ping pong analysis, Piaget's writings about concept formation showed that we can see ***that*** a ping pong stroke is produced by combining separate streams of object, movements and intentions, but we cannot see ***how*** it is produced. Our concepts are formed by "juxtaposition" but not by "synthesis." In the psychological examples, the problems were located in a ***critical moment*** of change that could be approached only indirectly. During such a critical moment, "multiple possible movements" change into "a single actual movement."

The new Quad Net model is constructed by modifying and combining aspects drawn from standard physics models. Standard physics models are defined as arrangements in ***space*** while new models are defined as arrangements in ***time***. Aspects are stated in four preparatory steps and that combine rational operations, physical momenta and material properties. Aspects and operations of the model are developed in a progressive series. Constructions are only partial but anticipate a completed construction.

As viewed from an anticipatory perspective of completion, an ***engineered organism*** is a body of material parts and devices operating according to a compact set of principles. The body produces movements that resemble those of an animal. Twitching "mover" devices resemble twitching muscle fibers. Each mover is driven by a bursting device ("burster") that generates signals in the form of "bursts of pulses." Pulses resemble nerve spikes, also known as "action potentials." Arrays of movers resemble muscles; and corresponding arrays of bursters resemble groups of neurons in spinal vertebrae that drive muscles. Bursters are organized by a network of "quadnet" devices in which collective bodies of devices exercise freedom. The current ***Wriggler*** project aims for a cylindrical muscle-like system similar to that of an eel or snake and would produce a repertoire of movements and related bodily feelings.

In approaching freedom, muscle-like movements are foundational. For example, in a model of vision, an "eye that looks at objects" locates objects in the external environment through operations of internal muscle-like movements that focus the lens of the eye on a visual edge. (See the timing device project "An eye for sharp contrast.") In a series of operations, the eye shifts from edge to edge and from object to object. Focusing and shifting movements of the eye detect the distance to an object and its spatial relations to other objects and the head that contains the eye — and these detections occur prior to any generation of images. Image-generation may take place in another layer of activity.

The third and fourth preparatory steps discuss unique properties of material bodies held under special conditions called the ***critical point***. The fourth and last preparatory step discusses the Ising model of mathematical physics that was developed to describe the critical point in magnetic bodies. At the critical point, clusters of Ising model elements change easily into each other; while, away from the critical point, clusters are relatively stable and unchanging. In a similar but more highly activated construction, quadnet models generate competing fragmentary movements that change easily into each other during a ***critical moment*** in a process that also leads to a final whole-body movement. Anticipated operations of quadnet models select movements and generate Shimmering Sensitivity, my proposed physical principle of freedom.

Certain quadnet operations occur cyclically. During each cycle, the collective body passes through a critical moment during which a single actual movement, if any, is selected. At the beginning of the critical moment, multiple possible movements co-exist as germinal fragments. Fragments may contest for control and may change into each other, giving rise to "Shimmering." Selection of one germinal fragmentary movement to become an actual movement may depend on very small influences, which is "Sensitivity."

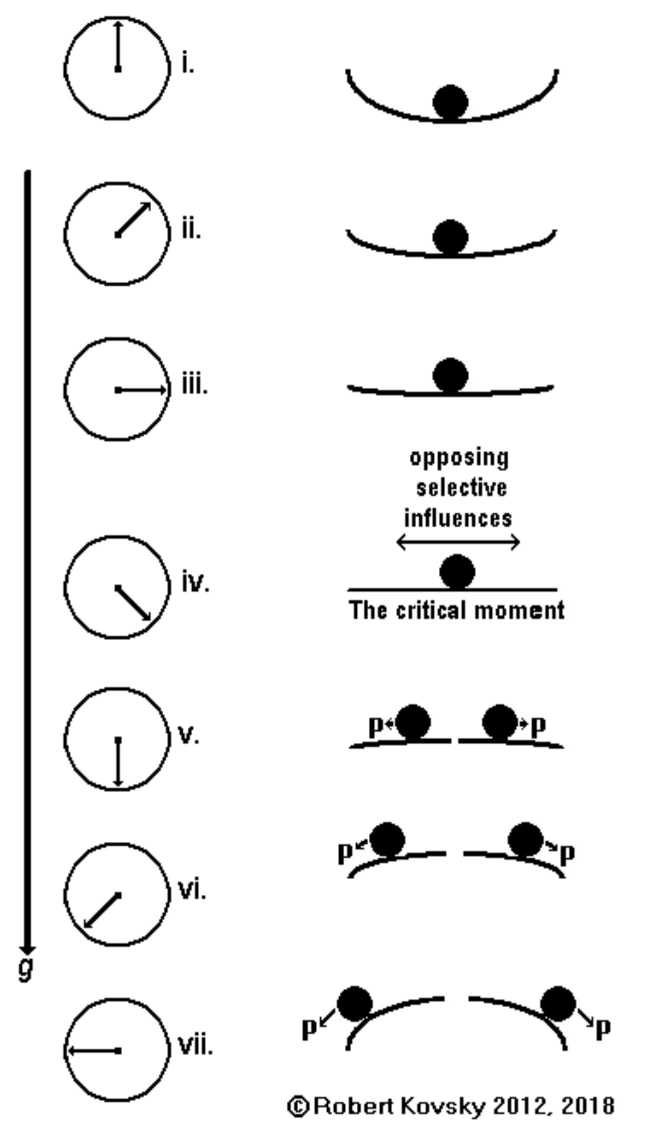
An organism has a large number of selecting components, e.g., quadnet devices in engineered organisms and neuronal groups in vertebrate animals. Selecting components are interconnected in large-scale networks. A large-scale condition of Shimmering Sensitivity pervades a network of components that pass through critical moments together. An influence incident at one component can affect the entire selection. During successful selections, the whole body produces coordinated movements that manifest integrity.

first preparatory step: dynamical selection in a gravitational field

A mechanical and dynamical ***process of selection*** is shown below in a succession of instantaneous images of a ball and a bowl, with tracking of time by a clock. During the process, the surface of the bowl under the ball goes through changes. As a result of the steady force of gravity **g** and momentary selective influences at the critical moment, the ball acquires momentum (**p**) in one direction or the other. Momentum = mass × velocity. "Dynamical" refers to changes in momentum. The figure below shows possible results in both directions. Repetitive cycles are the same until the critical moment, with divergent possibilities afterwards.

As in psychological choices, selective influences decide the resulting movement during a critical moment in a transformational process. Here, the process includes a deadline. By clock-time (vii), the direction of movement should be clear.

Mechanical metaphor: dynamical selection



Movements of the ball occur under the influence of the steady gravitational force, **g**; but gravitational force has an effective strength that depends on the shape of the bowl-like surface. At the start, gravitational force has a stabilizing tendency. Next, the bowl flattens until, at the critical moment, the effective gravitational force falls to zero, ceasing to influence movements. Simultaneously, selective influences change an initial stable position of the ball into the final directed form of movement. Next — the selection acquires momentum from gravitational force. Steady gravitational force goes through a cycle of effects: stabilizing–null–pulling.

Selective influences are independent of gravitational force and can vary in strength and direction. Changes in selective influences may or may not be coordinated with the process that changes the surface of the bowl.

There may be a brief moment, just after the critical moment, when the direction of movement of the ball can be reversed by means of a steady influence in the opposite direction. Then, the selection becomes irreversible. The resulting direction of the ball can be used to influence other movements; or the direction might be coded and recorded for future uses. The speed of the ball at a certain moment in the cycle might be recorded and used to compare the influences.

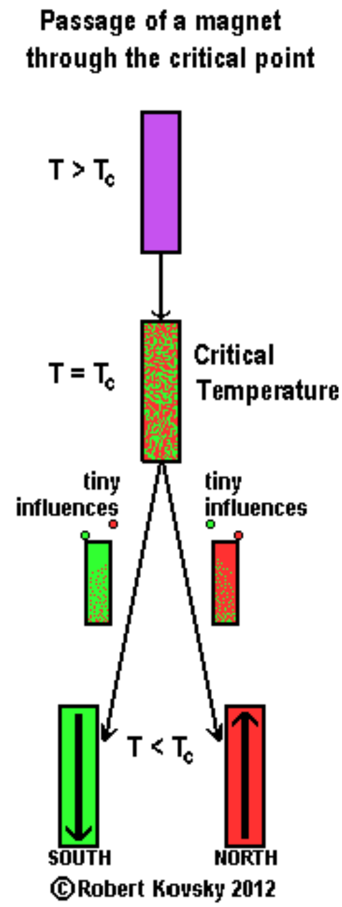
Finally — the initial position at clock-time (i) is reproduced for another cycle.

A ***balancing principle*** prevents actual movement prior to the critical moment but imbalance thereafter takes control. Balance is first held and then lost, leading to a selection and to action. Two independent shifting balances are combined. One shifting balance is based on the gravitational field and the changing bowl. The other shifting balance is based on opposing and variably unbalanced selective forces or influences during the critical moment.

There are two independent and distinct ***causes*** for the selection. The ***driving cause*** is the changing bowl-like shape in the overall process that runs from clock-time (i) to clock-time (vii) and that also imposes a deadline. During that process an underlying energy configuration goes through changes and these changes drive the selection. ***Selective causes*** arise from influences at the critical moment at clock-time (iv) that may be slight or variable and that may have no relationship to the driving cause. In this dynamical system, the momentum acquired by the ball depends on both the driving cause and the selective cause. A strong steady selective cause will start the ball off with a larger speed, compared to that produced by a weak, oscillating selective cause. The ball with the faster start will lead all the way down and the lead will increase as the cycle proceeds.

second preparatory step: a magnetic body passes through its critical temperature

The adjacent figure shows a process of cooling of a hot iron magnet through its ***critical temperature***, $\mathbf{T_C}$, also called "the curie temperature" or "the critical point." $\mathbf{T_C}$ is sharply defined as an exact number for each physical material, e.g., iron, cobalt or nickel magnets. If the temperature, **T**, of a magnet is higher than its $\mathbf{T_C}$, the magnet has no magnetic polarity.




In the process shown in the figure, as the magnet cools through its critical temperature, it acquires either a "North" or "South" polarity from "tiny" local magnetic influences. ***Susceptibility*** measures the effect of an external magnet on the polarity of a magnet being studied. At or just below the critical point or critical temperature, a magnetic body has an "infinite initial susceptibility;" meaning that a very small external magnetic field is sufficient to induce a polarity. At the critical point, the induced polarity is weak and can be reversed by a reversal of the external field. At lower temperatures, the polarity is strong and fixed. Colors and textures in images are suggested by the Ising model discussed below.

Other kinds of cooling processes that pass through the critical point provide results called the ***spontaneous magnetization*** (no external field or magnet) and the ***saturation magnetization*** (a strong external field or magnet).

Suppose that a cool magnet has a strong fixed polarity. If a polarized magnet is heated, it will lose its polarity at $\mathbf{T_C}$.

In a commercial application of the induced magnetization process, each "dot" in a ***magneto-optical memory device*** behaves like a separate body of magnetic material and is set by an external magnetic field during a cooling process, after being heated past the critical temperature by a laser.

The induced magnetic process resembles prior psychological and mechanical selection processes. I suggest that there is a ***class of critical selection processes*** that pass through a central "crisis" during which transient influences can become decisive and a choice is made between possible final results, leading to one actual final result. During the crisis, a transformation occurs. A critical selection process is a chief feature of freedom discussed below.

One important feature of magnetization processes is that gain or loss of polarity is ***sharp***. Suppose that a cold magnet holds a number of thumbtacks. If temperature rises steadily, the magnet becomes weaker and a few thumbtacks fall off one by one. Suddenly, in a brief moment at the critical temperature, all the rest fall off. The cooling scenario, e.g., in a spontaneous magnetization process, is the reverse of the heating scenario. As the temperature falls below the critical point, many thumbtacks suddenly fly into the air to attach to the magnet; then others follow one by one as cooling progresses. The number of thumbtacks measures the magnetic strength or magnetization of the magnet. As discussed below, such a sudden change in external effects corresponds to "sharp switching" in internal polarity. Sharp switching is another feature of freedom that is discussed below.

There are distinctions between the magnetic critical process and the dynamical critical process. In the magnetic process, selective influences and conditions of magnetism are ***states*** rather than movements. A magnetic state of iron can be prolonged indefinitely so long as the temperature is held at a fixed value — in contrast to a moving ball with a deadline and a finite range of motion.

A thermodynamic condition that can be prolonged indefinitely, e.g., a magnetic polarity and strength, is called ***an equilibrium state***. The magnetic process is made up of equilibrium states; the process can stop and start or proceed faster or slower without changing results. There is no deadline in a magnetic process.

In my approach, ***process*** is a general term based on continuity that includes both sudden dynamical movements and also indefinitely-prolonged states, as well as starts, stops and transitions between states. In contrast to the general concept of process, an equilibrium state is a special "null" kind of movement or change — somewhat like a zero in arithmetic. Magnetic processes have a ***quasi-static*** form consisting of short steps with pauses between steps that can be brief or prolonged. If pauses are very brief, steps may appear to join together in a continuous succession, like apparent movements in an animated cartoon. However, distinctions remain. In the dynamical mechanical process, the ball acquires momentum; but, in a quasi-static process, there cannot be any momentum.

Another important feature of the magnetic process is that the magnetic body is a material body with actual properties that are characteristic of an, e.g., iron material; these are measured during laboratory experiments. The mechanical process is a math-like construction premised on axiomatic laws of physics involving "gravity," "momentum" and "constraints." Material properties and math-like constructions are discussed separately in the next two steps as "natural science" and "rational science" approaches to phenomena seen at the critical point.

third preparatory step: natural science of critical point phase changes

Laboratory experiments show that many different kinds of simple materials, each maintained at or near an individual “critical point temperature,” manifest highly unusual features that are also “the same” or closely similar in all the materials. This fact is called ***universality***. The sharp change in polarization of a magnet is an example and illustrates features found at a critical point.

In this step, the previous critical point process for magnets is generalized to a larger class of ***critical point phenomena***. And even more generally, critical point phenomena belong to a still larger class of phenomena observed in material bodies that are called ***phase changes***, also called ***phase transitions***. During a phase change, ***the whole body is transformed***. E.g., liquid water freezes into ice or boils away as steam. Methods of cooking, pottery and metallurgy use controlled heat to cause whole-body transformations. Concepts of “phase change” are applied to many diverse phenomena. Phase changes in liquid crystal clock displays are caused by electrical signals. Changes in rhythmic patterns can sometimes be interpreted as phase changes, e.g., changes in the gait of a horse or changes in musical meter (march, waltz, 4/4). Insects pass through phases of egg, larva, pupa and adult. Stock exchanges have alternating bull markets and bear markets.

As stated by an important principal investigator, Sir Cyril Domb, in *The Critical Point: A historical introduction to the modern theory of critical phenomena* (1995) at 7-8, “thermodynamic properties of matter in equilibrium … fall into two groups: those which are smooth and continuous and those which have sharp discontinuities. As examples of the first group, we may cite the properties of ideal or nearly ideal gases (energy, entropy, specific heat, equation of state); of ideal and nearly ideal solids…. The second group is usually associated with phase transitions of various types: liquid-vapour equilibrium and the critical point, the melting of solids ….order-disorder transitions in alloys, ferromagnetism…”

Conventional scientific methods focus on smooth and continuous properties of matter, tethered to equilibrium conditions, (conserved) energy, entropy and equations of state. My alternative approach is based on phase changes, discontinuities and dissipative virtual energy (VE) operations in dynamical non-equilibrium domains that differ foundationally from conventional methods.

I suggest that conventional methods apply only partially to phase transitions of material bodies under ideal equilibrium conditions and fail to account for more highly activated movements of animal bodies and proposed Virtual Energy devices. Supporting observations are provided by physicist David Ruelle:

“One puzzling phenomenon is the boiling of water, and the freezing of water is no less mysterious. If we take a liter of water and lower the temperature, it is not unreasonable that it should become more and more viscous. We may guess that at low enough temperature it will be so viscous, so stiff, as to appear quite solid. This guess about the solidification of water is wrong. As we cool water we see that at a certain temperature it changes to ice in a completely abrupt manner. Similarly, if we heat water it will boil at a certain temperature, i.e., it will undergo a discontinuous change from liquid to water vapor. The freezing and boiling of water are familiar examples of ***phase transitions***. These phenomena are in fact so familiar that we may miss the fact that they are strange indeed, and require an explanation. ... So here is a problem for theoretical physicists: prove that as you raise or lower the temperature of water you have phase transitions to water vapor or ice. Now that's a tall order! We are far from having such a proof. In fact, there is not a single atom or molecule for which we can mathematically prove that it should crystallize at low temperatures. These problems are just too hard for us." (*Chance and Chaos* (1991) at 122-124 (emphasis in original).)

Critical point phase transitions are a special kind of phase transition. In a closed container, liquid water and water vapor can co-exist and change into each other (evaporating and condensing) over a range of temperatures that goes from below 32°F. to 705.4°F., depending on pressure. In other words, phase transitions occur at all temperatures in the range. The critical point phase transition occurs only at the specific temperature of 705.4°F. It is different from all of the other phase transitions between liquid water and water vapor, which form a continuous spectrum. At the critical point, discontinuous features are in the fore.

There is a voluminous scientific literature on critical point phenomena and related mathematical models, including sources for this project that were written by pioneers in the field: Domb, *The critical point*, supra; L. P. Kadanoff, *Statistical Physics: Statics, Dynamics and Renormalization* (1999); H. E. Stanley, "Scaling, universality, and renormalization: Three pillars of modern critical phenomena," Rev. Mod. Phys., Vol. 71, No. 2, Centenary 1999, S358-S366, available online. (Stanley suggested the thumbtack imagery used in the second preparatory step.)

In this step, features of critical point processes in material bodies are extracted from experimental evidence. Features are further developed in connection with the mathematical Ising model in the fourth step. After importation into the different Quad Net domain, features extracted from critical point phenomena will be developed into new principles of freedom.

Chief features of critical point physics and their uses are:

a. universality and scaling— justifying both importation in general and also to suggest principles of coordination and resemblances that might operate in QN assemblies;

c. plunge to zero and sharp switching — imported to suggest Sensitivity;

d. long-range order and critical opalescence — imported to suggest Shimmering.

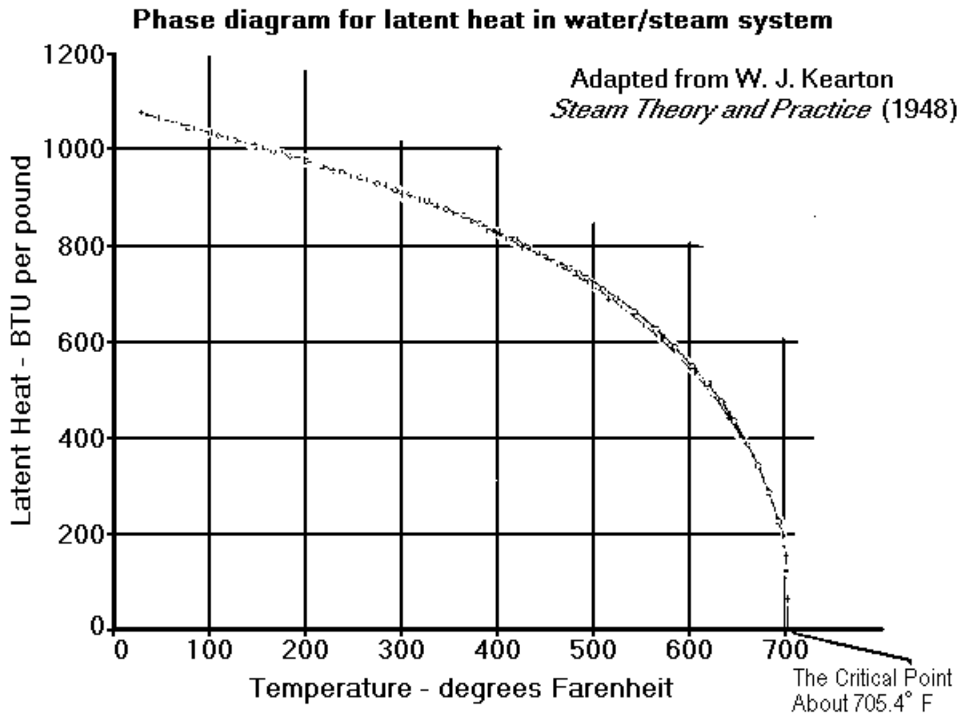
**a. universality and scaling.** Critical point ***universality*** is based on laboratory experiments showing that a large number of diverse physical systems display closely similar or ***common features***: e.g., scaling, plunge to zero and long-range order, along with intensively-investigated "critical exponents," which are outside the scope of this project. Physical systems displaying such universality include magnets made of various materials, liquid-vapor systems (e.g., $H_2O$, $CO_2$), various mixture of liquids, and various metal alloys (e.g., beta brass). Engineers incorporate critical point features in new materials (e.g., liquid crystal displays).

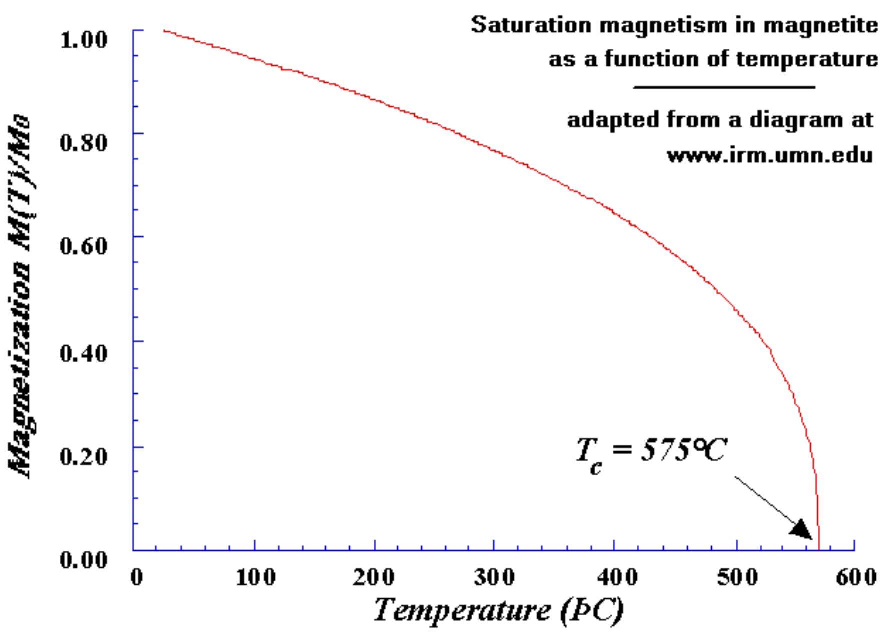
Common features in $CO_2$ and magnetism were noted in 1895 by Pierre Curie, who investigated critical point magnetism in iron. (Domb, 12-14, 82.) Common features suggest common underlying principles manifested in all the various materials. Mathematical models discussed below can share such principles with physical systems; that is why the Ising Model is useful. Insight into one system — whether material or mathematical — can provide insight into all.

The concept of ***scaling*** has multiple aspects. A grocery scale assigns weight numbers to material bodies. A mathematical scalar such as "temperature" assigns numbers to points in space. There is also "large scale" vs. "small scale." In graphs called ***phase diagrams*** shown on the next page, divisions of the x-axis and y-axis have a "scale" made of numbers and spacings. By means of "scaling," axis scales are stretched and squeezed to suit various purposes, e.g., to show universality.

In a phase diagram, many measurements or points are organized in a graph. There is also often a ***trace***, that appears to be continuous and to connect points. However, each measurement is a separate event. In conserved energy thermodynamics, bodies are generally presumed to conform to an ***equation of state***: this means that measuring the body twice under "the same" external conditions — typically the same temperature, pressure and/or magnetic field — produces "the same" results regardless of intervening events. Such bodies can be maintained in equilibrium. Living animal bodies, in contrast, often change on their own, e.g., when waking up, and do not conform to an equation of state.

Common large-scale features are shown below in graphs of latent heat in the water/steam system and saturation magnetization in iron magnetite. As discussed below, latent heat is stored in steam during evaporation; similarly, magnetic energy or “magnetization” is stored in magnetite during cooling. The two trace curves manifest “the same” shape over the whole range of temperatures. Both culminate in a discontinuous “plunge to zero” at the critical point. Spacings and numberings along axes of the diagrams have been adjusted to show similarities in the traces. Each point in a graph identifies a separate measurement at a fixed temperature.

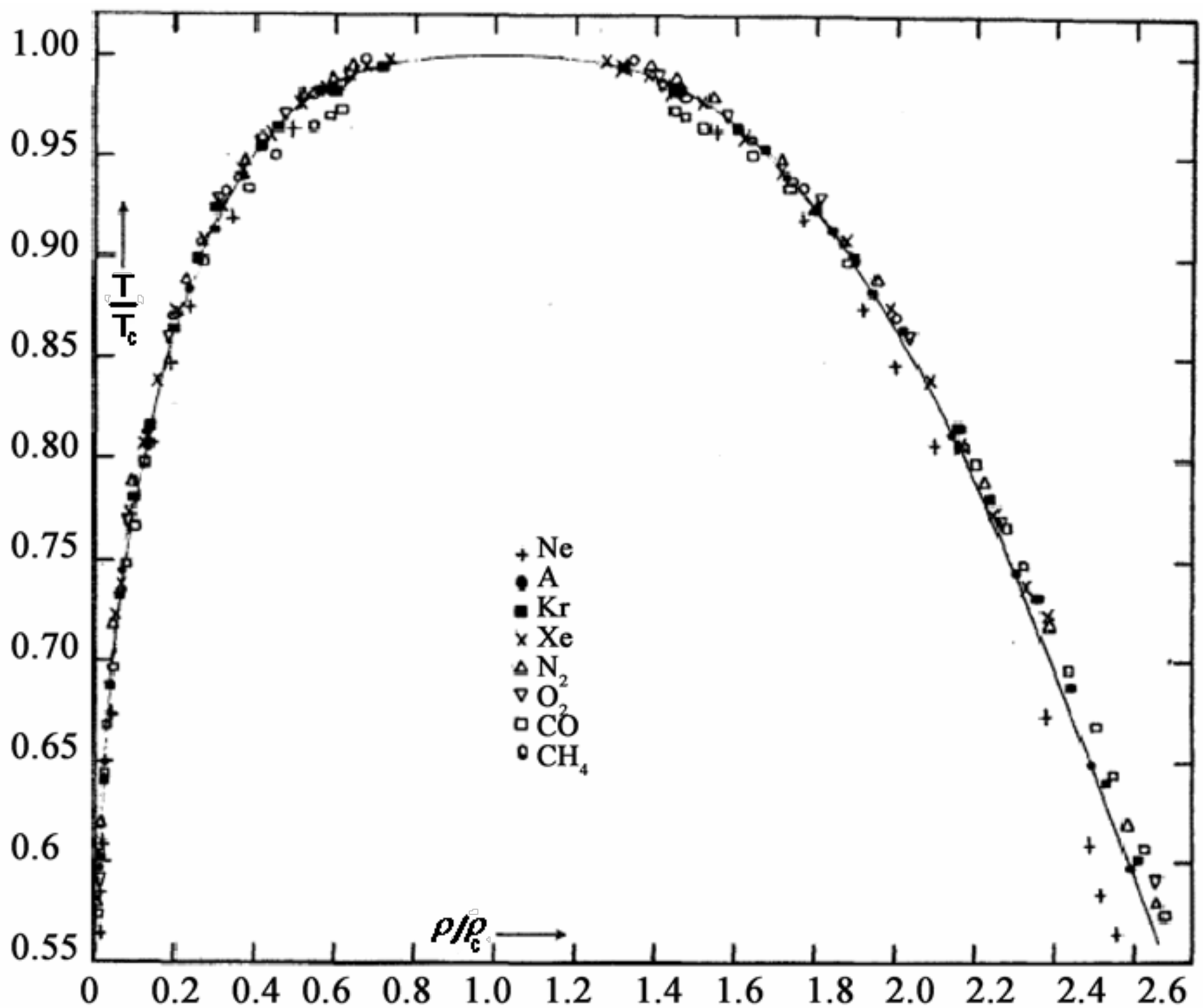


In the diagram below, measurements of eight different chemical reagents extend over full ranges of values and are superimposed to demonstrate common features. As stated by Domb at 21, the figure shows "Reduced densities of coexisting liquid and gas phases for a number of simple molecular fluids (Guggenheim 1945)."

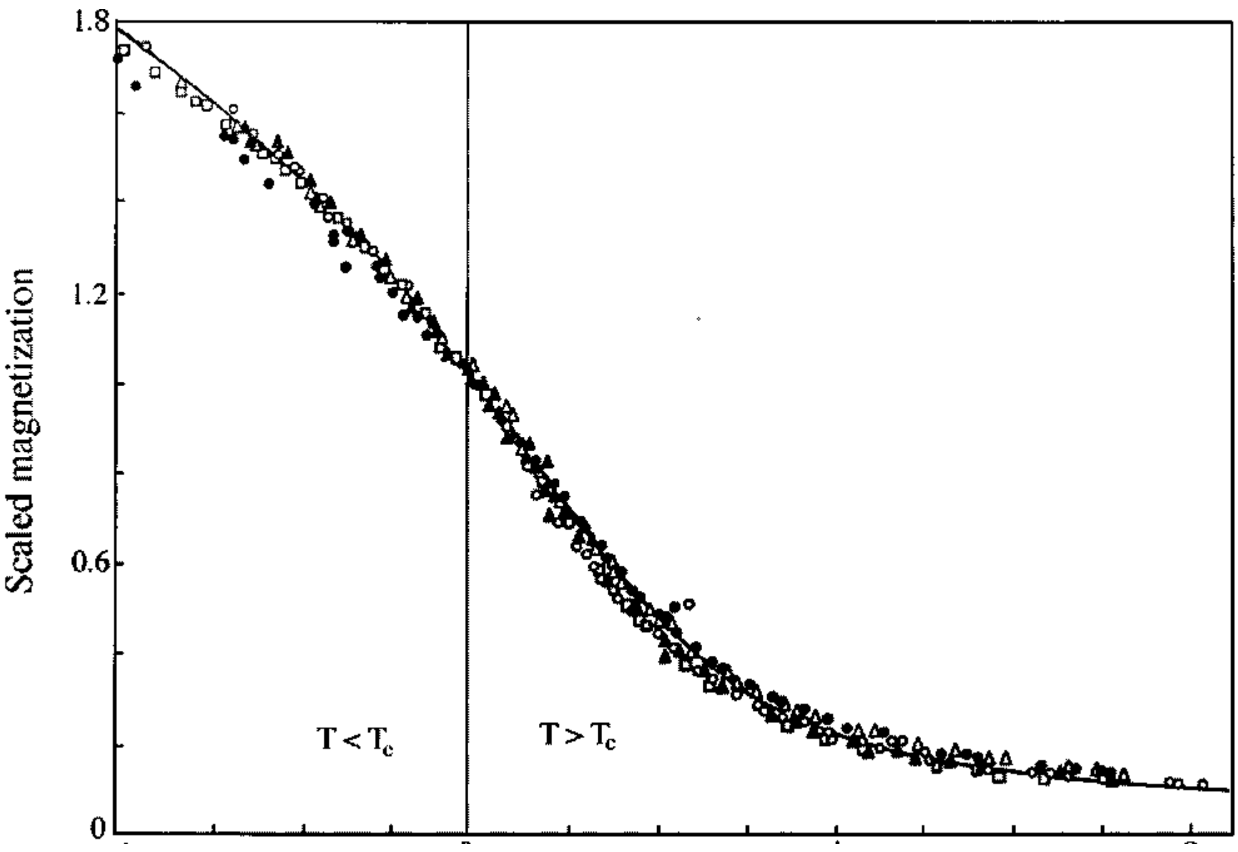
"Reduced" means that data points are re-organized around the critical point. Density of material, ρ, is compared to $\rho_c$, the density at the critical point, which is at (1.0, 1.00). Temperature is compared to $T_c$, the temperature at the critical point.

Data points are supplemented by a speculative line connecting the points. There is an absence of data at the critical points of the various materials. It is very difficult to maintain and measure fluid systems close to the critical point. (Domb, 203.) Away from the critical point, a slight disturbance of equilibrium is speedily resolved by processes called "relaxation." In contrast, very close to the critical point, hours or days may be required for the material to return to an equilibrium condition. As discussed in my article "A Patchwork of Limits: Physics Viewed from an Indirect Approach" (2000), at the critical point, "the system never relaxes."

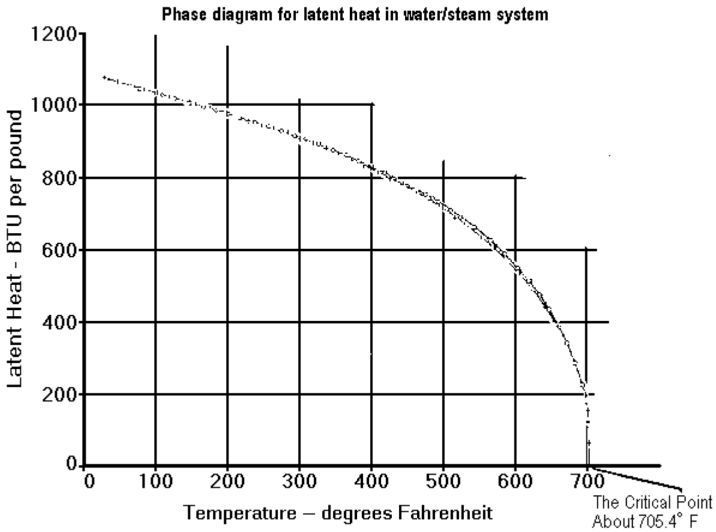


Previous images have shown (a) similarities between properties of magnets and water/steam and (b) similarities involving properties of different kinds of liquid/vapor systems. The image below from Stanley shows similarities involving properties of five different kinds of magnetic materials. In these measurements, each magnet is subjected to an external magnetic field (H) and the resulting magnetic strength or magnetization (M) depends on temperature (T).

Stanley's caption states: "Experimental *M H T* data on five different magnetic materials plotted in scaled form. The five materials are $CrBr_3$, EuO, Ni, YIG, and $Pd_3Fe$. None of these materials is an idealized ferromagnet. [Stating individual characteristics.] Nonetheless, the data for all materials collapse onto a single scaling function…"



**c. plunge to zero and sharp switching**




The phase diagram for latent heat in the water/steam system, shown in a reduced size in the adjacent image, illustrates the features of plunge to zero and sharp switching.

Latent heat was identified by Scottish professor Joseph Black (1728-1799) and applied by his friend James Watt (1736-1819) in the development of steam engines.

Each point in the phase diagram represents measurements of water in a closed container maintained at a certain pressure and at that certain temperature that is the boiling point. When pressure is increased, water boils at a higher temperature.

It takes 180 British Thermal Units (BTU) of heat energy to raise the temperature of a pound of water from 32° F. to 212° F at atmospheric pressure. This is a definition of BTU. As shown in the phase diagram, it takes an additional 970 BTU to convert that pound of hot water into steam while temperature remains at 212°F. This additional energy is called the "heat of vaporization" or ***latent heat*** that is stored in steam and that is partially recoverable in a steam engine as mechanical work (e.g., running a pump) when the steam condenses back to water.

The critical point of $H_2O$ occurs at 705.4° F. and with a pressure of 3206 pounds per square inch (psia). (Pressure measurements are compared to a vacuum and denoted "psia." For purposes of comparison, atmospheric pressure is 14.7 psia.)

At temperatures below the critical point, liquid and gas separate from each other and a ***surface tension*** shapes blobs of liquid water into aerial drops and puddles on surfaces. At the critical point, latent heat plunges to 0 and surface tension similarly plunges. (Domb, 74.) Liquid and gas interconvert and interpenetrate each other at all scales.

As temperature rises above the critical temperature, the gas becomes increasingly chaotic. The critical point identifies the transition or cross-over between a chaotic regime and a regime ordered by division into liquid and gas. At the critical point, exactly at the transition, an intermediary condition appears that has unique properties of distinct but thoroughly interpenetrating phases, with a highly complex and highly variable boundary between them.

At a lower pressure of 3000 psia, the boiling point of water is 696° F. and the latent heat is 213 BTU per pound, again indicated in the diagram. Hence, by shifting the temperature by less than 10° F. at high pressure, more latent heat has been stored in the steam than is required to heat ice water to boiling hot over the range of 180° F, holding at atmospheric pressure. At 696° F., water and steam are clearly distinct. Just a short step away is the onset of chaos. This is a high level of sensitivity and "sharp switching" based on the plunge to zero and discontinuity at the critical point.

**d. long-range order and critical opalescence**

Any successful phase change manifests long-range order. E.g., if a billet of hot metal alloy is properly quenched, the resulting material will be whole and strong.

At the critical point, however, whole-body changes have a different character. As shown by infinite initial susceptibility discussed earlier, at the critical point, the polarity of a whole body can be reversed by tiny changes in an external influence. Mathematics provides additional guidance, as discussed in the next step. In his review article, Stanley asks "why do we care" about critical point phenomena — to answer the question, he develops a mathematical description of long-range order: "A third reason is awe. We wonder how it is that spins 'know' to align so suddenly as $T \longrightarrow T_c^+$. How can the spins propagate their correlations so extensively throughout the entire system that [magnetization] ≠ 0 and [susceptibility] —> ∞?"

Mathematical descriptions suggest that long range order accounts for a laboratory phenomenon called ***critical opalescence***, where light shining through a fluid system at the critical point shows ever-shifting, whole-body variations. The phenomenon was first seen in the 1860's, when Andrews made systematic observations of carbon dioxide gas and liquid. According to Andrews' original paper, when the critical point was reached, "the surface of demarcation between the liquid and gas ...disappeared. The space was then occupied by a homogeneous fluid which exhibited ... a peculiar appearance of moving or flickering striae ["bands"] throughout its entire mass." (Domb at 97.) Domb writes: "It is indeed striking to observe a colourless transparent fluid suddenly becoming opaque and changing colour in a narrow band of temperatures around $T_c$." (*Id.*)

There are also limitations to long-range order in critical point phenomena, even in laboratory experiments with specially prepared materials and bodies. Critical points in vapor-fluid systems are difficult to maintain or measure. In sizeable magnets, multiple forces break up domains of magnetic uniformity.

fourth preparatory step: rational science of the Ising model

The original "Ising model" was a rational construction that was investigated by Wilhelm Lenz (1888-1957) and his student Ernst Ising (1900-1998). A rational construction is built using mental images, axioms and mathematical methods. Some rational methods have a rigid and narrow integrity called "rigor." Other rational methods allow for inventive variations. Families of Ising models are investigated with rational methods called ***statistical mechanics*** that has rigorous ways to construct core concepts and that also requires opportunistic inventions.

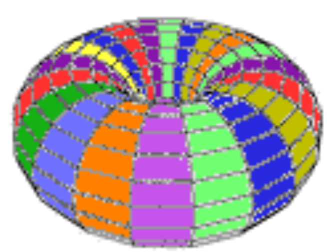
An Ising model consists of an array of elements in cells, also called a lattice of spins at points. An array or lattice has collective properties called ***magnetization*** and ***internal energy*** that are intended to model laboratory measurements of actual magnets. Higher internal energies correspond to higher temperatures. Here, simple square Ising models aim to describe critical point magnetization processes. An array of n rows and n columns contains a total of N cells where $N = n^2$.

Features of the Ising model form are shown in the adjacent image of example 1. An element in a cell has one of two polarities: (1) "spin-up," denoted with (red) **O**; and (2) "spin-down," denoted by (green) **X**.

Two cells in an array are ***neighbors*** if they are in the same row and in adjacent columns — ***or*** if they are in the same column and in adjacent rows. In the model, the last cell in a row is a neighbor to the first cell in that row; and the last cell in a column is a neighbor to the first cell in that column. This arrangement gives rise to a ***toroid***. The adjacent image shows a flat 20×20 cellular form with red and green elements and also a 20×20 toroidal surface with colors added for clarification. Flat forms are convenient for Ising models but toroids reappear with different details in quadnet devices.

**Ising model example 1**

| | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| X | O | X | O | X | X | O | O | X | O | X | O | X | X | X | O | X | O | X | O |
| O | X | O | X | X | O | O | X | O | X | O | X | X | O | O | X | O | X | O | X |
| O | O | X | O | O | O | X | O | X | O | X | X | O | O | X | O | X | X | O | O |
| X | O | X | O | X | O | X | X | O | X | O | X | O | X | X | X | X | O | O | X |
| O | X | O | O | O | O | X | X | O | O | O | X | X | X | O | X | O | X | X | O |
| X | X | X | O | X | O | X | X | O | X | X | O | O | X | O | X | O | X | X | X |
| O | O | X | X | X | O | O | X | X | O | O | X | O | O | O | O | O | X | X | O |
| O | X | X | O | O | X | O | X | O | X | X | O | O | X | O | X | O | X | X | O |
| O | O | X | O | X | O | X | X | O | O | X | O | O | X | O | X | O | O | X | X |
| X | O | X | O | X | O | X | X | O | X | O | O | X | O | X | O | X | X | O | O |
| O | O | O | X | X | O | O | O | X | O | X | O | O | X | O | X | O | X | X | O |
| X | X | X | O | X | O | X | X | O | X | X | X | O | X | O | X | X | X | X | X |
| O | O | X | O | X | X | O | O | O | X | X | O | X | X | O | X | O | O | X | O |
| O | X | O | O | X | O | X | X | X | O | X | O | O | X | O | X | O | X | X | O |
| O | X | X | X | X | O | X | X | O | X | X | O | X | X | O | O | X | X | O | X |
| O | X | X | O | X | O | O | X | O | O | X | O | O | X | O | X | O | O | X | O |
| O | X | X | O | X | O | X | X | O | X | O | O | X | O | X | O | O | X | O | X |
| O | X | O | O | O | X | O | X | O | X | X | O | O | X | O | X | O | X | X | O |
| X | O | X | O | X | O | X | X | O | O | X | X | O | X | O | X | O | O | X | O |
| O | O | X | O | X | O | X | X | O | O | X | O | O | X | O | X | O | X | X | O |



In a general form, there is a set of n×n Ising models. In an ideal Ising model, n approaches ∞. However, all sizes share common features and important features will be apparent in 8×8 versions discussed here. The following construction models a process of spontaneous magnetization and there is no external field.

Ising model ex. 2 shown below, a "random" arrangement, illustrates calculations of magnetization and internal energy in an 8×8 Ising model.

Let R denote the number of red elements in the array and let G denote the number of green elements. R + G = N, where N = 64 for an 8×8 array. Define the ***magnetization*** of the array as $\mathcal{M}$ = M×μ where M = R – G and where μ is the magnetization of a spin-up element and –μ is the magnetization of a spin-down element. In other words, the construction models the different strengths of a magnet ($\mathcal{M}$) by flipping plus and minus elements and by changing the collective proportion of elements. M can be positive or negative. If all the elements are red, the polarity is spin-up and the strength is at its greatest or R = N = M; and a similar result is obtained with contrasting details when all elements have a green, spin-down polarity: G = N = – M . If R and G are equal, the magnetic strength $\mathcal{M}$ = 0.

The Ising model has a collective "internal energy" quantity denoted by ***IE*** and stated in integral units of J. Rules for calculation are stated by Kadanoff (214): "there is an interaction between neighboring spins which gives a lower energy to spin configurations in which spins are aligned" and (in the equation for the Hamiltonian) each aligned neighboring pair of spins reduces the energy by one J.

In ex. 2, the number of red elements in each row is counted and entered as a partial sum at the end of the row under the black R heading. Addition of partial sums provides R, the total number of red elements. G = 64 – R and $\mathcal{M}$ = μ×(R – G). In ex. 2, $\mathcal{M}$ = 8μ, expressing a preponderance of red elements.

Energy units are counted for each row and each column and entered as black partial-sum "e's" in the furthest right column and the bottom row. Each pair of aligned neighbors adds one energy unit. Last elements are neighbors to first elements. Energy units in columns are added to those in rows to include all alignments and the resulting sum is denoted as E and called the ***energy count***. Each partial-sum e is an even number. In ex 2, the energy count is E=62J.

**Ising model ex. 2**

```
                R
X O O X O X O O 5 2
O X O X O X X O 4 2
O X O X O O X O 5 2
O O X O O X X O 5 4
X X O X O X X O 3 2
O X O O O X X O 5 4
O O X X O O X O 5 4
X O X O O O X X 4 4
4 4 4 2 8 4 6 6   e
 Total R=36, G=28
    M=8μ, E=62J
```

The relationship between E and $\mathcal{IE}$ is based on an energy scale that is defined by endpoints shown in the two adjacent arrays, Ising models ex. 3 and ex. 4. In ex. 3, every element is aligned with all of its neighbors and the energy count is E=128J. In ex. 4, there is not even one pair of aligned neighboring elements and the energy count E=0. Call ex. 3 a "uniform array" and ex. 4 a "checkerboard array."

The calculation rule from Kadanoff stated above "gives a lower energy to spin configurations in which spins are aligned." Hence, the array in ex. 3 has the lowest possible energy. Conversely, the array in ex. 4 has the highest possible energy.

Define the internal energy of the array in ex. 3 as the ***minimum internal energy***, denoted as $\mathcal{IE}_m$; and define the internal energy of the array in ex. 4 as the ***maximum internal energy***, denoted as $\mathcal{IE}_M$. The definition is: variable internal energy $\mathcal{IE} = \mathcal{IE}_M - E$, where E is the energy count. E.g., $\mathcal{IE}_m = \mathcal{IE}_M - 128J$. Because E is always an even number, the energy scale has no more than 64 steps. As an example, the internal energy for ex. 2 is $\mathcal{IE} = \mathcal{IE}_M - 62J$. The value of the baseline $\mathcal{IE}_M$ is arbitrary, similar to baselines of energy scales in other mechanics paradigms.

```
Ising model ex. 3
                R
O O O O O O O O 8 8
O O O O O O O O 8 8
O O O O O O O O 8 8
O O O O O O O O 8 8
O O O O O O O O 8 8
O O O O O O O O 8 8
O O O O O O O O 8 8
O O O O O O O O 8 8
8 8 8 8 8 8 8 8   e
  Total R=64, G=0
    M=64μ, E=128J
```

```
Ising model ex. 4
                R
X O X O X O X O 4 0
O X O X O X O X 4 0
X O X O X O X O 4 0
O X O X O X O X 4 0
X O X O X O X O 4 0
O X O X O X O X 4 0
X O X O X O X O 4 0
O X O X O X O X 4 0
0 0 0 0 0 0 0 0   e
 Total R=32, G=32
           M=0, E=0
```

There is one other array that has the same internal energy as that in ex. 3: namely, an array with all elements uniformly spin-down and green. Likewise, there is one other array that has the same internal energy as that in ex. 4: namely, an array where every green element is replaced by a red element and every red element is replaced by a green element, which is a checkerboard with colors reversed.

Let's count the number $\mathcal{N}$ of all the possible arrays in an 8×8 Ising model, with every possible arrangement of red and green elements. This number is $\mathcal{N} = 2^{64}$, which is extremely large. This number of arrays is distributed over no more than 64 steps in the energy scale. The number of arrays is 2 at each of the endpoints. Hence, at a step in the mid-range of that scale, there must be an extremely large number of possible arrays.

Arrays in examples 5a, 5b and 5c all have the same energy count of E=86J while magnetization shifts from 24µ to –24µ. Arrays share the first, third, fifth and seventh rows, which are all uniform and fixed. The second, fourth, sixth and eighth rows have variable divisions that allow for shifts in the magnetization. Flipping an element at the edge of a division changes the magnetization by 2µ without changing the energy count. The three arrays have the same partial sums e.

```
Ising model ex. 5a
               R
O O O O O O O O 8 8
X O O O O O O O 7 6
X X X X X X X X 0 8
X O O O O O O O 7 6
O O O O O O O O 8 8
X O O O O O O O 7 6
X X X X X X X X 0 8
X O O O O O O O 7 6
4 4 4 4 4 4 4 4   e
Total R=44, G=20
M=24µ, E=86J
```

```
Ising model ex. 5b
               R
O O O O O O O O 8 8
X X O O O O O X 5 6
X X X X X X X X 0 8
X X X O O O O X 4 6
O O O O O O O O 8 8
X X X X O O O X 3 6
X X X X X X X X 0 8
X X X X O O O X 3 6
4 4 4 4 4 4 4 4   e
Total R=31, G=33
M= -2µ, E=86J
```

```
Ising model ex. 5c
               R
O O O O O O O O 8 8
X X X X X O X X 1 6
X X X X X X X X 0 8
X X X X X X O X 1 6
O O O O O O O O 8 8
X X X O X X X X 1 6
X X X X X X X X 0 8
X X O X X X X X 1 6
4 4 4 4 4 4 4 4   e
Total R=20, G=44
M= -24µ, E=86J
```

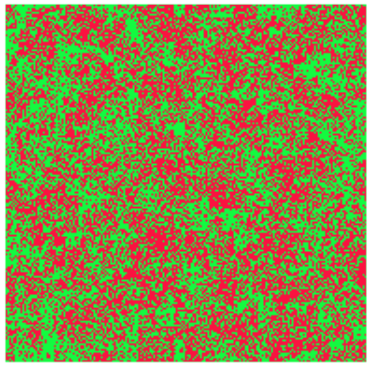
Arrays in examples 3, 4 and 5 suggest the form shown on the next page that resembles forms for critical selection processes that were discussed in prior psychological and physical examples. In the new form, a high internal energy (*IE*) array, with a high temperature, has an underlying checkerboard texture with some flips and patches. In contrast, a low *IE* array, with a low temperature, has an underlying texture of uniformity with some patches and flips. In both cases, there are "some" deviations from the underlying pattern. The extent of the deviations depends on the temperature. Deviations can be located anywhere in the array.

At the endpoints of the process, where there is perfect uniformity or a perfect checkerboard, each flip raises the *IE* by 4J, at least until flipped elements become neighbors. If temperature is at either extreme, it would take 64 flips to change one checkerboard into the other checkerboard with reversed polarities or to change one uniform array into the other uniform array with the opposite polarity.

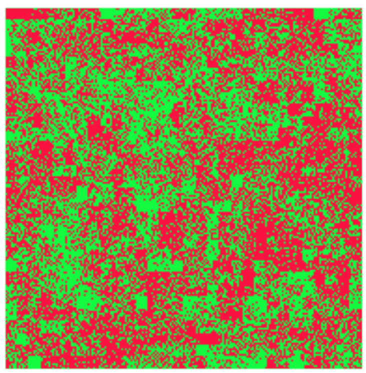
In between the energy extremes, towards the middle region, there are an enormous number of possible arrays and some can be arranged in incremental shifting patterns like those in ex. 5.

Thus, while magnetization is fixed at each energy extreme, it is highly variable in the middle.

Based on the foregoing, four Ising model arrays with a larger size are arranged below in the pattern of a critical selection process. Each of the four arrays is "typical" of arrays that conform to the rules of the Ising model at the indicated temperatures, although "typical" may be of dubious meaning at the critical point.



At a high temperature, a "griddy" texture that resembles checkerboards appears on multiple scales. Red and green elements are equally balanced.

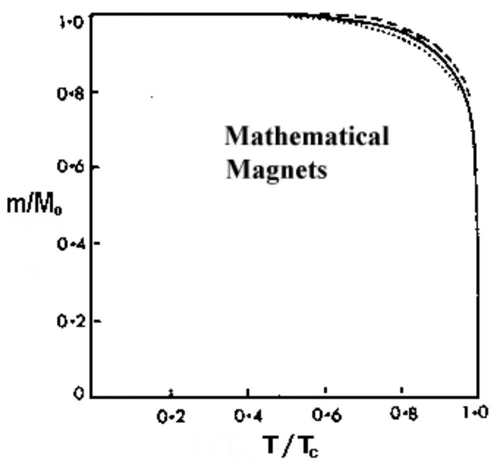


At the critical temperature, clusters of elements have variable shapes, textures and interconnections that may appear detached from the underlying grid. Rough balance continues between red and green clusters but it varies a lot and can be quite rough over periods of time.

At a low temperature, one polarity or the other achieves a dominance or consensus. Deviant elements are spread out. Each possible condition is highly unbalanced.

Methods of statistical mechanics provide substantial insight into critical point patterns in Ising models. At each temperature or specific value of E, there is an ***ensemble*** of possible arrays. Each array in an ensemble is equally possible, according to the "fundamental hypothesis of equal *a priori* probabilities." (R. C. Tolman, *The Principles of Statistical Mechanics* (1938), at 59, 350. Other authors have different but similar fundamental hypotheses as noted by Tolman in the footnote at 65.)

An ensemble is organized by a ***distribution*** of possible arrays; in other words, the possible arrays for each temperature can be counted and compared. Values are calculated using formulas that incorporate the distribution. When the n×n size of arrays is increased to "infinity," traces that connect points show discontinuities at the critical point while values at lower temperatures fit into smooth curves. The plunge to zero in natural materials has a parallel in the Ising model, as shown by the graph below, adapted from Domb, Fig. 5.12.




The 3 curves in the graph show properties of Ising models of an infinite extent with honeycomb, square and triangular lattices. Magnetization **m** is defined with respect to a low temperature standard $\mathbf{M_0}$; temperature **T** is defined with respect to the critical temperature $\mathbf{T_C}$. The plunging curves measure magnetic strength at variable temperatures and the three curves for the three lattices are closely similar, thus illustrating universality and scaling.

Note that the plunge to zero in the graph of the Ising model is even steeper than in graphs of laboratory phenomena. The Ising model is a two-dimensional system while natural materials are three-dimensional. Three-dimensional versions of the Ising model appear to be mathematically "intractable" (insoluble) or nearly so.

Sensitivity based on the plunge to zero in the Ising model resembles that discussed in connection with latent heat. At the critical point, "no energy" is needed to flip between polarities or between a polarized condition and a non-polarized condition. Not far below the critical point, a lot of energy is needed to flip the polarity.

The critical point in the Ising model resembles the focal critical moment in the mechanical process of step one as to a balancing principle that operates "prior to" something critical, then turning into a loss of balance that operates "afterwards." However, "prior to" and "afterwards" are metaphorical in statistical mechanics, in contrast to the dynamical mechanics of step one where "prior to" and "afterwards" are actual stages in a process that changes momentum.

While recognizing their value, I suggest that statistical mechanics and the Ising model have serious shortcomings. In Ising model ex. 5 above, each array requires a separate calculation and nothing in the calculations puts ex. 5b "in between" ex. 5a and ex. 5b. The order of arrays in ex. 5 is arbitrary. The critical process pattern is likewise arbitrary. Only certain particular arrangements (among many possible arrangements) show "prior to" and "afterwards." There is not even a "quasi-static" process to connect the images as there was to connect points of laboratory measurements and to construct their traces.

Starting with a given 8×8 array, there are 64 ways to flip an element to get to a new array; and then there are 63 ways to flip an element to get a third array and so forth. The possibilities quickly become too complicated to unravel and they accumulate in a heap (also known as a "partition function" or Tolman's "sum-over-states"). Many different organizational schemes are imposed on the heap for particular purposes. It is the organizational schemes that suggest the patterns.

Shortcomings were evident when James Clerk Maxwell, Ludwig Boltzmann and Josiah Willard Gibbs invented mechanical and statistical thermodynamics. Their axiomatic distributions involved ***probabilities*** rather than possibilities. Each set of probabilities exists in a kind of imaginary universe with an enormous number of dimensions, perhaps $10^{23}$, which is even larger than $2^{64}$. Boltzmann, for example, proposed an "ergodic theory" where points of system variables in such a universe are visited in a connected way and periods of time spent by the system at the various points are apportioned according to the probabilities.

Tolman discusses a different approach, the ***fluctuation theory***, "a somewhat approximate but important practical method of treating the fluctuations" originally developed by Marian Smoluchowski (1872-1917) and Albert Einstein (1879-1955). Fluctuation theory is said to account for the blue color of clear sky. According to Tolman, it also accounts for critical opalescence. "For a fluid in the neighborhood of its critical point the fluctuations predicted by [a particular equation] would become very large… This accounts in a qualitative way for the striking appearance of opalescence in an illuminated fluid as we approach the critical point, the scattering centers now becoming very important and large enough to scatter light of longer wave-length than the blue." (Tolman at 647.)

Kadanoff has a different approach to fluctuations. The axiom of energy conservation applies to the statistical "mean" or average of the probabilities. Another statistic is the "deviation from the mean," which measures whether the probabilities are closely clustered around the mean or are spread out in a broad range of different values. First, Kadanoff introduces an equation involving a "fluctuation." "The fluctuation is, of course, a statistical quantity in that it refers not to averages but to deviations from the mean." (At 16.) Next he derives an equation in which "the energy fluctuations are related to a very familiar thermodynamic quantity." Then: "The interested reader might now ask, what fluctuations? If one holds a bottle of water in ones hand one knows that the energy of the water has a very well defined value. So what is [the equation] doing talking about fluctuations? The answer is simple. A statistical mechanical system does have fluctuations in energy and essentially all measurable quantities." (At 16-17.)

Whatever the approach, fluctuation theory makes it possible to organize the heap of Ising model arrays according to transitions between arrays by means of flips and according to relative probabilities of transitions. In an array at a temperature extreme, such as in ex. 3 or ex. 4, a fluctuation produces one flip and the flip changes the internal energy (*IE*) by 4J. Hence, it is supposed that the fluctuation must contain at least 4J of energy to accomplish this task. In the arrays of ex. 5, a fluctuation produces one flip and the *IE* does not change. Perhaps the *IE* changes while the flip is going on but it takes a lot less energy than 4J. In a probabilistic domain, flips that require less energy are "exponentially more likely" than flips that require more energy.

Looking at the situation from another angle, flipping an element in ex. 5 is like flipping a cellar door that is held partially upright by a frame as compared to one that is flat on the ground, as in ex. 3 and ex. 4: the slanted design makes flipping easier. At the critical point, the frame is vertical and the door flips easily.

Therefore, in the Ising model, a lot more flipping occurs in the mid-range of energies than at energy extremes. In the mid-range, flipping can occur "spontaneously" and in a collective way.

The fluctuation model for critical opalescence also leads to the culminating feature of ***long-range order*** that is unique to critical-point systems, where it is "universal." (Domb, 117-118, 131-132, 143-45.) In the Ising model, short-range order is a "nearest-neighbor interaction" where each spin or element interacts only with spins or elements that are adjacent to it in a lattice or array.

Starting with a system at equilibrium, suppose that a local fluctuation or flip is introduced. This changes the likelihood of a flip at a different location. The ***correlation range*** at a particular temperature measures the region of interactive influence of a fluctuation: the greater the correlation range, the wider the region of influence. The correlation range varies with temperature. When flips are rare and isolated from one another, e.g., at temperature extremes, there is no correlation. When flips are frequent and occur in neighboring elements, collective changes can occur and correlation ranges can become large. Rows, columns and patches can flip together. Of chief importance, fluctuation theory suggests that correlation ranges grow to infinity as the temperature approaches the critical point. Then, long-range order governs.

"That the density fluctuations will get large near the critical point was pointed out by Smoluchowski, but that in addition the correlation of the density fluctuation at different points will become of very long range and that this is the cause of the critical opalescence was first clearly stated by Ornstein-Zernike in 1914." (G. E. Uhlenbeck, "The Classical Theories of the Critical Phenomena" in *Conference on Phenomena in the Neighborhood of Critical Points Held at The National Bureau of Standards Washington, D.C. April 5 to 8, 1965* at 16.)

Long-range order means that a unit's correlated or coordinated activity with a distant unit is about as strong as with a nearest neighbor. Because the only interaction between units is purely local (i.e., nearest-neighbor), this effect is astonishing. ***Local interactions unite to form global or collective activity***.

Long range order is based on a balance between (1) one exponential factor that reduces a unit's influence as distance from it grows and (2) a second exponential factor where a unit's influence grows with distance based on the number of interaction paths between any two units. "Right at the critical point, the gently decaying power-law correlation factor in the number of interaction paths, previously negligible, emerges as the victor in this stand-off between the two warring exponential effects. As a result, two [magnetic] spins are ***well correlated even at arbitrarily large separation***" (Stanley at S365, emphasis added).

My 2006 *Quad Nets* article referred to underlying psychological and physical problems of integration and juxtaposed Stanley's remarks with leading questions posed by researchers in neuroscience:

> Correlation at arbitrarily large separation recalls a principle (or problem) in neuroscience, the ***binding principle***. Edelman & Tononi at 106-107 write: "When we see a scene, we are not aware of colors, movements, and forms separately and independently but bind the color with the shape and the movement into recognizable objects. ... Binding, for example, assures the integration of the neuronal responses to a particular object contour with its color, position, and direction of movement. .... This binding principle...is repeated across many levels of brain organization." The principle is confirmed by imaging and EEG studies that show widely-separated brain centers are activated together. (Posner & Raichle.) In considering proposed "mesoscopic" activity (at scales of activity larger than neuronal), Freeman (372-73) states that "brain imaging of metabolic activity and cerebral blood flow [shows] involvement of large swatches of cerebral cortex in conjunction with diverse tasks ... The question is, how might mesoscopic domains become coordinated and form the macroscopic domains that may underlie the swatches."

## C. Quad Nets and Shimmering Sensitivity

**Overview**. The anticipated Quad Net model suggests a new approach to Freeman's question on the previous page about coordination of brain domains of multiple sizes. The approach here is more of a circumnavigation at a middling altitude than a royal road to the summit but it touches many bases and views the summit from multiple perspectives.

The Quad Net model combines features previously discussed: (1) a dynamical character like the mechanical process of the first step; (2) properties of actual materials, like those shown by the iron magnet in the second step and by materials discussed in the third step; and (3) rational principles of universality, balancing and collective selection that have been abstracted from critical point investigations of material bodies and the Ising model. The features are reconstructed as physical principles of freedom and integrity in the anticipated Quad Net model.

The Quad Net (QN) model also has features that are new and different. "Virtual Energy" principles impose constraints on designs and operations without determining outcomes. "Virtual Energy devices" have dissipative operations fueled by a constant input stream of Virtual Energy (VE) and they never reach equilibrium. The proposed condition of Shimmering Sensitivity is new and unique.

Primal QN designs resemble the Ising model but instead of spin patterns arrayed in space, phases in the quadnet model are patterns of pulses arrayed in time. Moving patterns of pulses in a quadnet device resemble previously-discussed states in an iron magnet or simple fluid. A ***pulse*** is an instantaneous transfer of energy between devices and serves as an idealized model for impulses or "spikes" in biological nerves, which are also known as "action potentials." The quadnet model is constructed from elements of time and action; cyclical quadnet processes involve changes in rates of pulsation and in timing intervals. The model is constructed around variable temporal forms that resemble music more than computations, e.g., in "A Tube for Transport," in the *Paradigms* project.

I suggest that the quadnet model can lead to multiple classes of interconnected devices in systems that produce complex movements of engineered organisms, similar to those of biological organisms. Some features of the model are speculative and others depend on presumed material properties. This approach permits imaginary designs with only a general concern for actual implementation. I suggest that, at least, the designs show ***how*** exercises of freedom are possible in an animal body, thus reaching closer to goals discussed in the Piagetian interpretation of ping pong in part A.3.

Discussion is simplified because designs start with simple “primal” devices. Each primal device is developed into a ***kit of parts*** containing multiple devices based on the primal device but performing more complex functions. The parts catalogue of VE devices resembles that of a robotics engineer. In anticipated designs, VE devices are interconnected in “tiled” assemblies and modules, typically in varying ways that depend on collective properties of materials; and modules are then further interconnected to make up large-scale ***engineered organisms*** that move like animals in purposeful ways.

My prior online publications have set forth designs for extensive kits of parts of pulsers, timing devices, force devices, bursting devices and quadnet devices. In current and anticipated designs, signals generated by bursting devices and by timing devices set and trigger movements of muscle-like force devices. Influential signals from “sensitive” timing device modules vary according to internal conditions, internal changes and external events; and I suggest that such signals resemble signals of internal bodily feelings and of perceptions of an external world.

In projected goals for designs, quadnet devices provide integration of movement and sensing, with operational assemblies at two levels: (1) at a lower level of bodily feelings and movements of spinal joints and eyes (similar to vertebral ganglia and to cranial nerves controlling eye movements) and (2) at a higher level of the whole body (similar to the cerebellum). Integration of movements occurs at the lower level but subject to control from the higher level. Still higher “cerebral” levels generate imagery of external events and objects to follow, imitate, etc.

In other words, anticipated designs employ two levels of control, one “residential” (in the joints) and the other “remote” (in the head), borrowing terminology from William James. A cylindrical engineered organism might resemble an eel with a head, a flexible and jointed spine and a muscle-like body attached to head and spine. Modules made of QN devices and bursting devices “reside” in each spinal joint and at roots of eye motor nerves; and modules collectively pass through critical moments of Shimmering Sensitivity that produce integrated movements from a repertoire of movements, with selections influenced by signals detecting and measuring conditions and events in the interior of the body, resembling influences of feelings, and by external objects on which eyes focus.

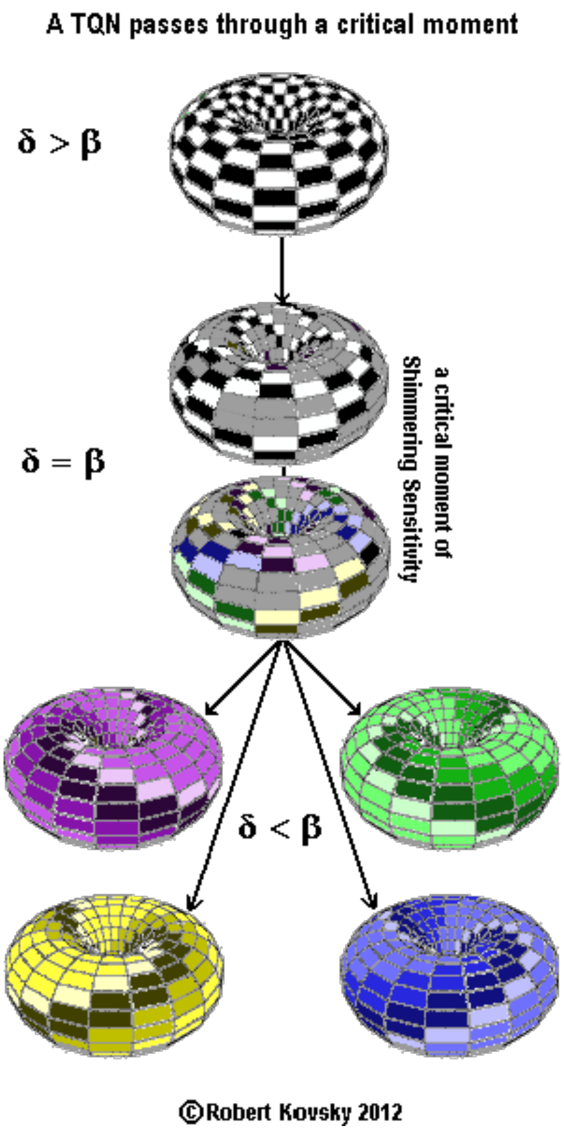
In addition, a larger assembly of QN devices and bursting devices is “remotely” located in the head and is organized as a “body map” of the residential bursters. The head also contains devices that process signals from exterior visual, auditory, tactile and chemical sensors. The head produces collective integrated signals. Bursters and QN devices in the body map in the head are connected to matching devices in each joint. Subject to ongoing movements at the joints and signals resident in the spinal layer, integrated pulse patterns generated in the remote body map in the head control movements of the body.

This project first develops one “primal” exemplar of a selection process in the Quad Net model and then expands the perspective to suggest development of collective arrays. Thus, the model of freedom starts with features of freedom appearing in a single QN device governed by ***elemental short-range-interactions*** and ***quasi-static operations*** that lack ***momentum***. Operations appear to be slow and integrity appears to be tenuous. Speed and integrity can be improved by addition of long-range interactions and by dynamical processes that involve momentum. Further improvements are based on presumed properties of ***material media***: an elastic body in which the quadnet devices operate, with an internal watery environment that serves as a substrate for an array of devices in a QN device and that supports ***entrainments*** of pulsations. I suggest that, in such an environment, a continuing condition of Shimmering Sensitivity can be maintained by means of arrays of interconnected QN devices that pass through critical moments together, some ***synchronously*** and others in overlapping waves. Flickering synchronous and overlapping moments of Shimmering Sensitivity in devices combine into a continuing flow and circulation with continual production of movements in response to sensations. Layers of quasi-static elemental operations, dynamical processes and material media unite to produce ongoing exercises of freedom and integrity of movement.

1. elemental Quad Net devices and operations
    a. overview of the primal Quad Net critical selection process

A critical selection process occurs in a ***Toroidal Quad Net*** or TQN, as shown at successive stages in images below. The TQN is a sheet of materials in the shape of the surface of a doughnut (torus) like the toroidal form in the Ising model; the primal form is made of elemental VE devices with nearest-neighbor connections. Each elemental device discharges a stream of pulses or a stream of bursts of pulses.

Changes in the selection process are tracked by two "timing intervals," δ and β, that collectively control operations of elemental devices.




The TQN can maintain stable pulse patterns when δ > β or when δ < β but the patterns are different. Pulse patterns at the critical moment when δ = β are unstable. Critical moment pulse patterns in the images are one example out of many possibilities while pulse patterns at endpoints are limited to a few variants.

The process starts with δ > β and with alternating discharges in a "checkerboard" pattern. The process ends with δ < β, where one of four wave patterns occupies the entire TQN. A wave pattern may manifest any one of several strengths, according to the number and timings of pulses in a cycle.

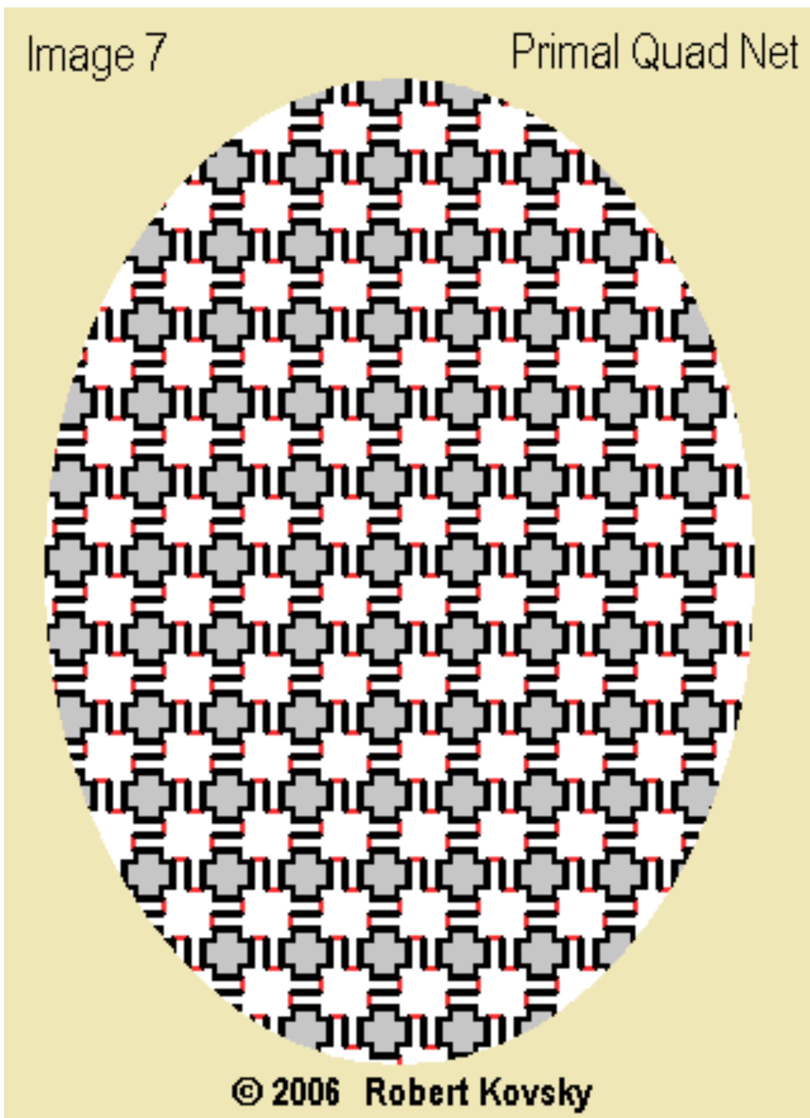
At the center of the process is a critical moment, when a choice is made that selects the final wave pattern. During a critical moment, multiple germinal wave fragments can compete in a contest for territory in the TQN. The outcome may depend on momentary conditions of the TQN or on very small external influences.

This TQN selection process resembles critical selection processes discussed in previous steps (psychology, mechanical metaphor, magnets, water, Ising model). I suggest that it unifies psychological and physical principles of choice.

b. Constructions are based on Primal Quad Net

Primal Quad Net (PQN) shown below is a quadratic array of ***elemental devices***. Elemental devices in a PQN operate according to axiomatic principles that are later augmented by additional aspects. "Quadratic" refers to 4-fold interconnections ("projections") rather than to square shapes. As illustrated shortly, operations are unchanged when devices and projections are stretched or squeezed.

The structure of PQN resembles that of the Ising model but elements are different. Each box-like elemental device in the PQN discharges ***pulses*** onto its nearest neighbors (note the red junctions between devices), causing some or all of such neighbors to discharge pulses. Elemental devices in a PQN originate as pulsers, timing devices or bursting devices. An elemental device is a model for a neuron and PQN is a model for neuronal tissue.

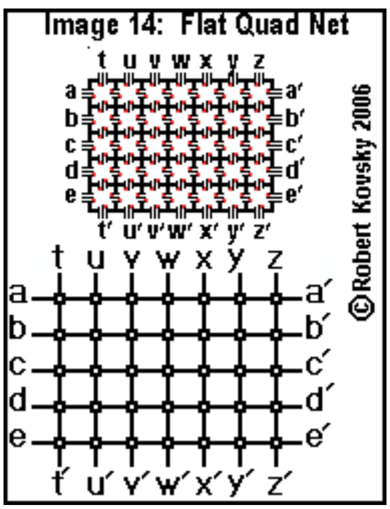


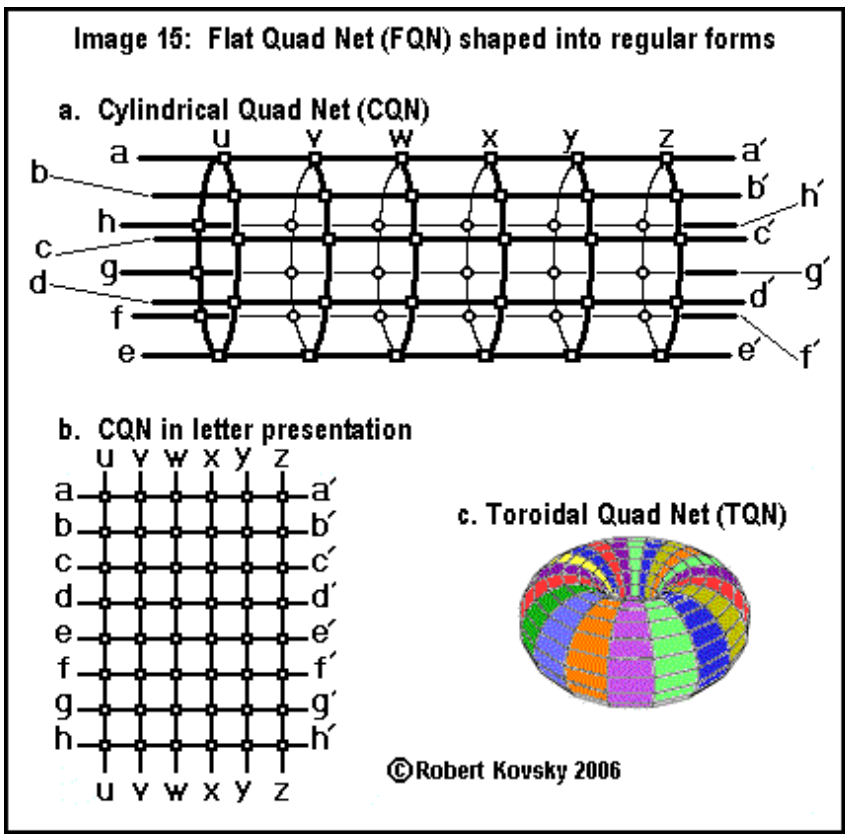
I suggest that elemental devices can be embodied in electronics devices. In further constructions, I suppose that PQN comes in the form of an elastic sheet of material holding a tiled array of elemental devices, in a pattern similar to wallpaper.

c. construction of the Toroidal Quad Net

Images below show construction of the Toroidal Quad Net (TQN) previously introduced. Image 14 shows a piece of Flat Quad Net cut from Primal Quad Net. Two versions in Fig. 14 illustrate equivalent designs. The lettering of projections along the edges identifies possible splices.

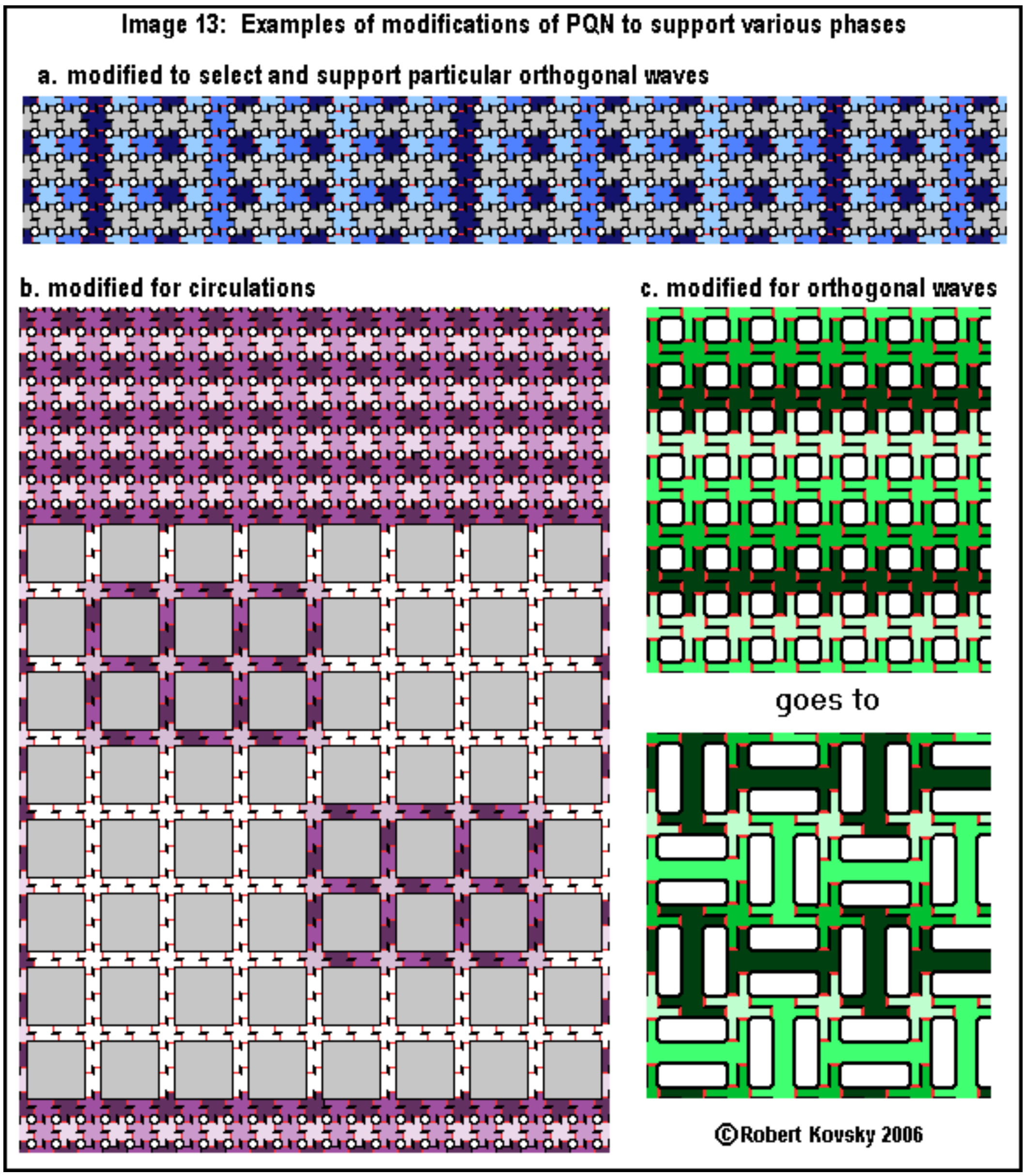
Splices are carried out in Image 15. Images 15(a) and (b) show a piece of PQN spliced to construct a Cylindrical Quad Net or CQN; two presentations are shown. Image 15(c) shows the exemplary TQN constructed from a square piece of PQN with 20 devices per row and column. First a Cylindrical Quad Net is constructed and then the ends of the CQN are spliced. In the finished TQN, interconnecting projections are only implied. Colors in the TQN of Fig 15(c) are used to clarify the structure rather than to show operations.



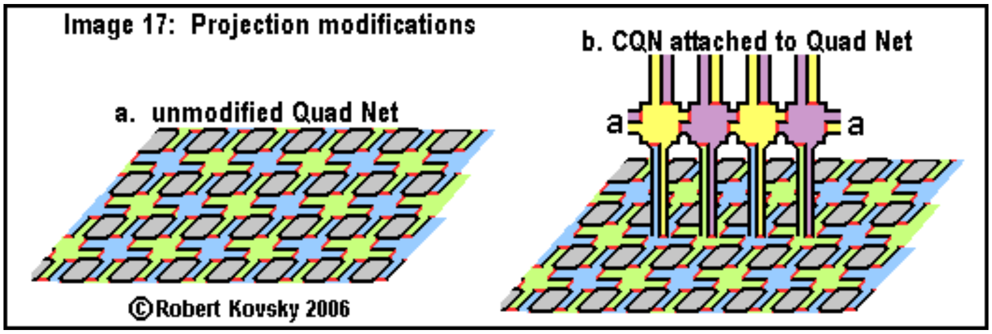



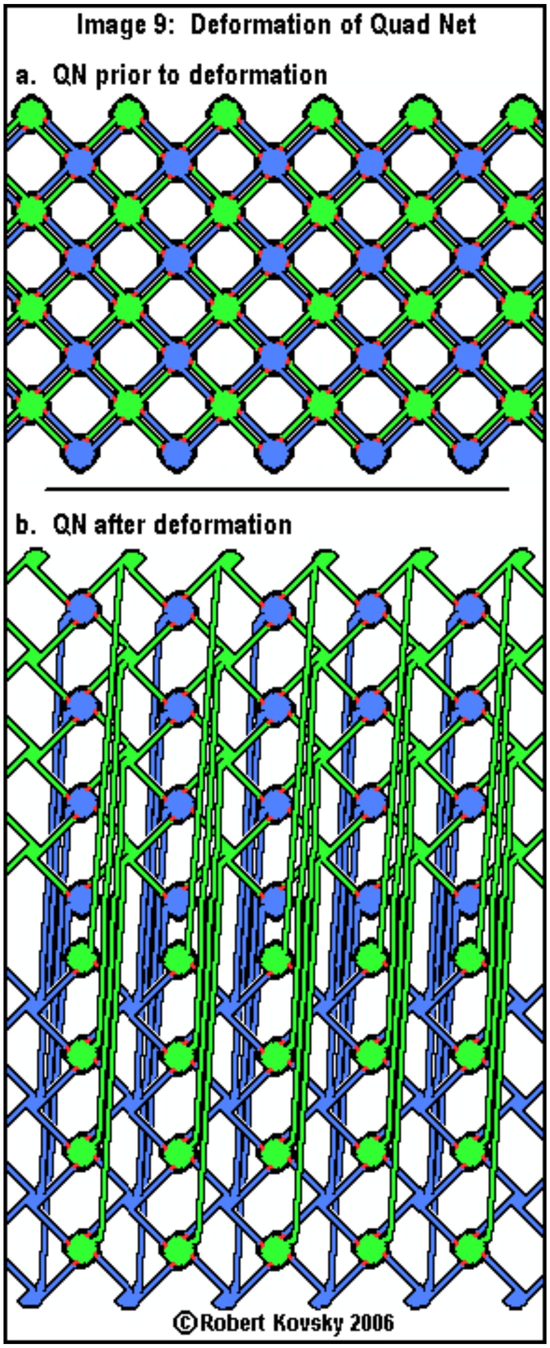
d. plasticity of designs using Quad Nets

Images below illustrate ways in which pieces of Primal Quad Net can be modified and deformed. Elemental devices in the examples in Image in 13 are “1 pulse trigger timing devices” – that is, a ready 1 pulse device will be triggered by the discharge of any of its nearest neighbors. Gray or blank areas are inactive. In modifications in Image (a), operating elements generate certain “orthogonal waves” with an apparent motion in one dimension. In Image (b), oscillating patterns in unmodified QN become circulating patterns in the modified portion. Shapes of elemental devices and their inter-connections are modified in Image (c), which carries orthogonal waves.



The image below shows splicing of projections from a Cylindrical Quad Net (CQN) onto a Quad Net (QN). In this design, elemental devices require 2 or more neighboring discharges in close temporal proximity in order to trigger a discharge. Signals from the CQN can influence a selection process in the QN when it is passing through a critical moment.






The Primal Quad Net in Image 9(a) has alternating colors added to elemental devices for purposes of presentation. The pattern is a checkerboard. This does not mean that the system is always checkerboarding, although that activity is in its repertoire.

In Image 9(b), projections are stretched and alternating elements are separated, a process of ***collective deformation***. A stretched projection more closely resembles a biological axon that ramifies close to other neurons.

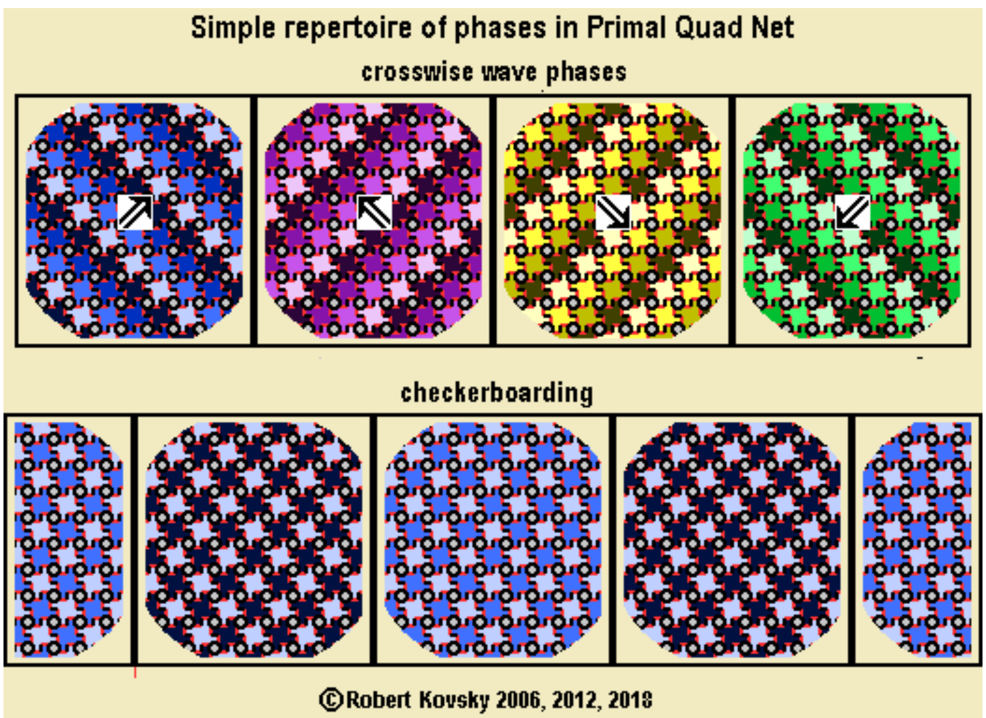
The deformations do not affect elemental operations. In further developments, it would be possible to modify one separated set of elemental devices by means of material variations. For example, in addition to simple checkerboarding, the separated collections of elemental devices can have reciprocating operations and balancing operations. Perhaps green-bodied elemental devices are immersed in a watery solution that affects operations while blue-bodied elemental devices continue to operate as they did before separation. Differences between signals reflect material variations.

e. pulse patterns or phases in PQN and a TQN; cycling and Shimmering

A PQN or TQN can maintain certain steady pulse patterns or phases. Also, dynamical processes drive transformations between phases that may be sensitive to influences and material properties. Some intermediate conditions are unstable.

The image below shows the repertoire (set of movements) of steady patterns or phases that can be maintained in a PQN or TQN with certain operating specifications that are called “2-pulse triggers,” meaning that two pulses from nearest neighbors in close succession are needed to trigger an elemental device. Wave patterns can have variable directions, speeds and strengths.

Each elemental device goes through a cycle of: (1) a period of ***charging***, followed by (2) an instant of ***triggering*** caused by neighbors’ pulses, followed by (3) a period of ***responding***; and (4) an instant of ***discharging*** through pulses, becoming “empty.” Then the cycle repeats. Conditions of an elemental device are shown in images by four hues that range from dark (empty) to bright (full and triggered).




For the checkerboarding phase, a 4-step cyclical process is shown at five moments (with repeating images truncated at the endpoints of the cycle). Two sets of interwoven devices alternately discharge. There is no direction of apparent motion.

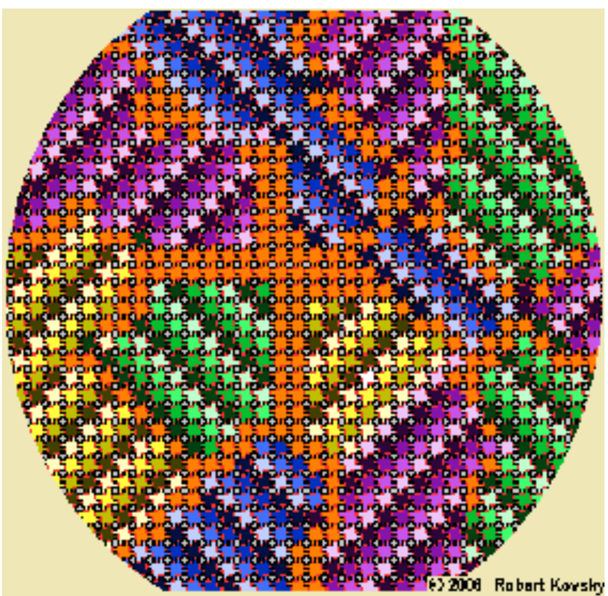
Wave phases, each shown at one specific instant, have four directions of apparent movement (shown by arrows), which might correspond to up-right-left-down. In each wave, dark devices have just discharged; their pulses have just triggered the bright devices, which are preparing to discharge. When bright devices discharge, they will turn into dark devices and all other devices will become one step brighter.

A precise statement of operations specifies variable ***timing intervals*** or periods of time between events in the cycle of operations of an elemental device.

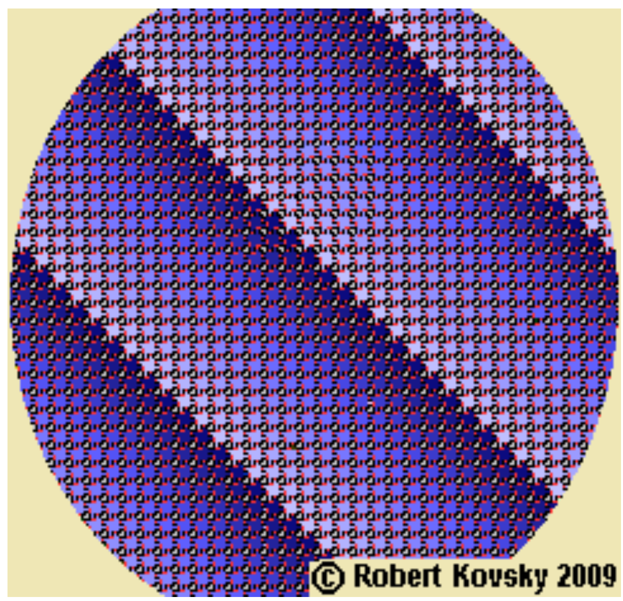
The timing interval **$\delta$** specifies the period of time between neighbors' discharges and the element's own discharge; it is called the ***responding period*** of the elemental device. In images of waves, changes in hue of an elemental device occur each $\delta$. $\delta$ is like the adjustable tick period of a metronome. The responding period and the number of elements in a cycle combine so that the period of a cycle of each crosswise wave phase shown in the images is $4\delta$.

The smallest cycle period of a wave is $(2^{+})\delta$ (a bit bigger than $2\delta$). This period occurs during the critical moment of a selection process when fragmentary wavelets are generated. In the figure below, a little later than the critical moment, $4\delta$ cycle pieces are used for clarity. The period of the cycles grows as the process goes forward; and little wavelets thencombine into a single wave that can be maintained in the whole Quad Net in a ***resolved*** condition, e.g., an extended wave as shown below. Ultimately the cycle period can extend over the whole Quad Net.

**A condition of shimmering in a QN**




**An extended wave in a QN**



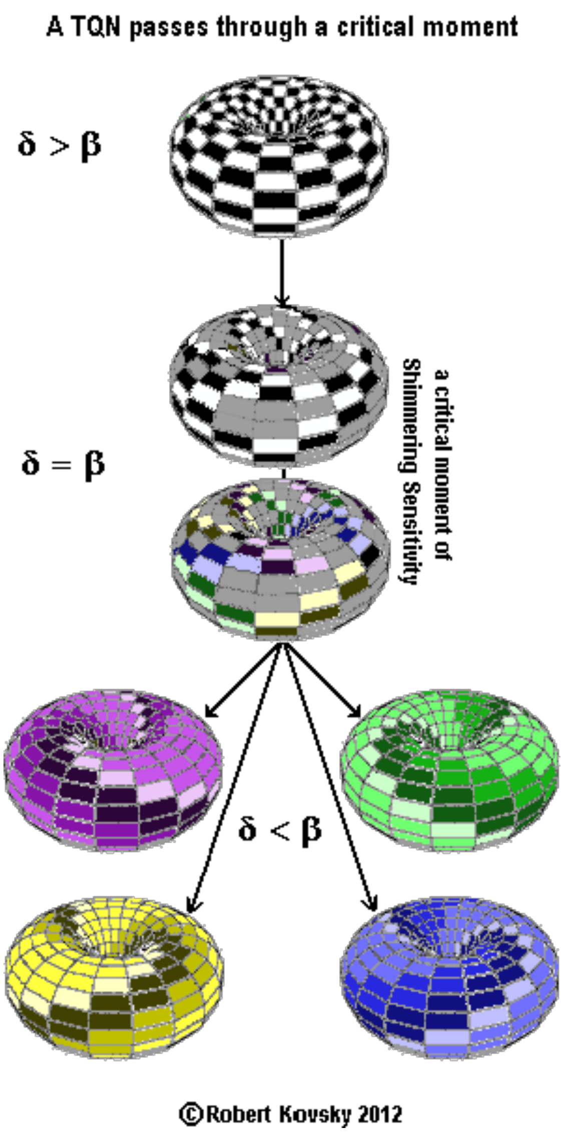

An element which has just discharged cannot be triggered; its condition is "refractory." The ***refractory period***, **$\beta$**, is the time after discharge of a pulse required for an elemental device to become ready to respond to new trigger pulses. When $\beta < \delta$, a device that discharges becomes ready to be triggered before its neighbors discharge; and the neighbors trigger it immediately. The result in checkerboarding. In waves, in contrast, a device is still refractory when discharge occurs in the neighbors it has triggered.

In this design, elemental devices in the Quad Net are called a "2 pulse trigger timing device," where triggering of the device requires arrival of two or more pulses within a timing interval $\xi$. For steady pulse patterns like those shown in the images at $\delta > \beta$ and $\delta < \beta$, $\xi$ is much smaller than the other operating variables, $\delta$ and $\beta$. It is possible to allow for greater mixtures of phases at the critical moment if $\xi$ is increased to a size comparable to $\delta$ and $\beta$.

f. the primal selection process in a Toroidal Quad Net

The primal critical selection process in the TQN can now be specified, at least in part. Elemental devices in the TQN are driven collectively through a process that starts with $\delta > \beta$ and that ends with $\delta < \beta$. To begin, suppose that $\delta/(\delta+\beta)$ changes steadily at a slow rate and that there are many pulsations during a critical moment.

While $\delta > \beta$, an elemental device that triggers neighbors will itself be triggered by those neighbors' next discharges, which will all occur at the same instant. Steady patterns appear in the form of alternating checkerboards. The smallest piece of PQN that can maintain checkerboarding is a square containing 4 elemental devices. The whole TQN does not need to pulse in unity but does so in the primal process for simplicity. Perhaps the initial condition is prepared by a researcher. Whether a patchwork or whole, each cyclical period of discharge is exactly 2δ.




At the other endpoint, when $\delta < \beta$, an elemental device that triggers neighbors will be in a refractory condition when those neighbor discharge. The four wave directions identify possible steady patterns.

At the moment shown for $\delta < \beta$, cycles have the maximum period of 10δ. The period of a cycle is variable. In this device, wave patterns can have a cycle period that varies continuously from $2^{+}\delta$ to 10δ. The change in cycle period from 2δ to $2^{+}\delta$ occurs when $\delta = \beta$ and specifies the critical moment when patterns shift from checkerboarding to waves according to external influences or material variations.

In wave patterns with period equal to 10δ, the TQN is occupied by 1 cycle (which is snaky, disappearing twice from sight). As many as 3 cycles can be fitted into this TQN if cycle periods are very small (close to $2^{+}\delta$) but that fit would be very tight. The larger the TQN, the greater the variations that can be produced in a wave phase. Generally, wave phases in a TQN vary in direction and also in strength and might be produced or measured by multiple variables.

2. primal features of freedom in critical selection processes are based on elemental operations of a Toroidal Quad Net.

**Preview**. Features of freedom were anticipated in previous steps. Discussion begins here with primal features of freedom manifested in elemental operations of a TQN. There are also limitations and shortcomings: initial applications are limited to slow, quasi-static processes; integrity is tenuous and chancy.

Later developments discussed below address limitations and shortcomings in the primal design. First, elemental short-range (nearest-neighbor) interactions can be augmented with ***long-range interactions***, e.g., between two reciprocating TQN's. Assemblies and collections of devices lead to new operations and functions, e.g., identical interconnected modules that circulate and maintain pulse patterns.

Second, dynamical drive processes connect ***momentum*** **(p)** of moving body parts to a critical selection process in a TQN; the resulting phase leads to a ***change in momentum*** (Δ**p**). A dynamical movement starts with a jerk (saccade) that can be prolonged and guided by means of a critical selection process that it drives. The initial saccade is reflexive but subsequent evolution can be scripted (habitual) or, in further developments, can be guided by rational processes.

In the third development, movements and patterns in collective assemblies (modules) also depend on suggested ***material properties*** of the module; I suggest that material properties influence selections and that these can be modeled through constructions in the nature of variable ***elastic strains, entrainments*** and ***training***.

Features of freedom are thereby organized in a layered construction where different layers contribute to an exercise of freedom according to the task and its circumstances and also according to past history of the organism, e.g., training.

The following primal features of freedom are stated in general terms so as to unify examples of psychology and operations of elemental Quad Net devices:

(A) During a central critical moment in a larger critical process in a body, balancing changes to loss of balance and multiple possible movements turn into a single directed movement.

(B) Critical processes can manifest sharp switching and/or sensitivity and can produce various steady, reproducible and/or reversible movements, but results depend on rates of processes during the critical moment.

(C) During a quasi-static critical selection process, a condition of indeterminacy arising during the critical moment may manifest a unique character that extends over the whole body and that leads to integrity of movement as the process progresses.

(A) During a central critical moment in a larger critical process in a body, balancing changes to loss of balance and multiple possible movements turn into a single directed movement.

The critical selection process in the TQN is the culmination of the critical selection processes discussed in previous psychological examples and physical paradigms. Such a critical selection process begins with an initial condition (state or movement) and concludes with a final condition (state or movement). The initial condition and the resulting final condition are of two different kinds and the initial condition turns into the final condition during a relatively brief critical event at the center of the process. In the TQN, similar to other cases, there is only one initial condition but there are multiple possible final conditions. The initial condition is balanced but the final condition is unbalanced. After the transformation is completed, the final condition is silenced; thereafter, a new initial condition may appear at a renewal of the cycle.

In prior cases, the following transformations occurred:

(a) from a condition of indecision to an actual decision in psychological examples;

(b) from a stationary ball to a moving ball in the dynamical metaphor;

(c) from an unpolarized piece of magnetic material to a polarized magnet; and

(d) from a checkerboard array to uniformity in the Ising model.

To this list please add:

(e) from a condition of checkerboarding to a wave in a Toroidal Quad Net.

In all the physical models, there is a feature called "symmetry" that changes during the process. In a static situation, symmetry means that two distinct aspects of a body are "the same" except for certain specified variations. The most important symmetry in actual life is that between the left side and the right side of the human body, especially as to whole-body movements. Similarly, a mirrored image has a symmetry relationship with the original. Different kinds of symmetries resemble different patterns in repetitive wallpaper or floor tilings. In an extension of the concept, identical bodies possess a collective symmetry where each body is "the same" as every other body in the collection.

In my approach, "the same" denotes a successful "matching operation." When two images match, they are said to "be the same." A mathematical equals sign denotes a matching operation with a constraint of strict exactitude. Many matching operations succeed while tolerating discrepancies and approximations. E.g., "these two heat engines have the same efficiency" or "you look the same as last year." Generally, constraints imposed on matching operations depend on tasks, situation and circumstances.  Often, people say one event is "the same" as another event and then change their minds.

In an iron magnet and Ising model, two final magnetic polarities (North and South) have a crucial difference that is lacking in the initial condition.  Similarly, four final wave forms differ in direction.  Differences between final conditions of a magnet or TQN are not present in the initial hot unpolarized magnet or the initial checkerboarding phase in the TQN.  Such differences mean that the final condition has "less" or "lower" symmetry than the initial condition.  The more differences, the less symmetry.

This feature was discussed by Domb at 122 in connection with the insight of Lev Landau (1908-1968).  "The ordered low-temperature phase always has lower symmetry than the disordered phase and its symmetry group contains elements which are not invariant under transformations of the higher-symmetry group of the disordered phase."

In the Toroidal Quad Net, the "higher-symmetry group" is the balanced and undirected movement of checkerboarding and the "lower symmetry group" has four unbalanced wave movements that manifest different directions and strengths.

The initial condition and the possible final conditions in cases (a) through (e) set forth above share another important feature.  Both initial conditions and final conditions are few in number and possible conditions are stable or fixed.  At the endpoints of a critical process, it is very difficult to change the condition of the body.  In contrast, during the central critical event, multiple possible conditions can co-exist, although in a germinal form, and they can change easily into each other.

In other projects, I suggest that features of freedom and critical selection processes manifested in TQN operations are projected by human beings onto a social domain and become features of ***contests*** that take place in actual life.  At the beginning of a contest, contestants are in symmetrical situations while at the end of the contest, there is one winner and one or more losers.  Which contestant becomes the winner depends on many influential factors; in an athletic contest, influential factors may include bodily capacities, training, personal efforts and cheers from the sidelines.

During the critical moment of  competition or suspense, there are many possible configurations and the apparent winner may shift back and forth.  I suggest that such features of freedom of contests have parallels with features of freedom in the Quad Net model.  Inside a TQN, during a critical event, there can be a contest or competition among the wave fragments for occupation of the Quad Net field.

(B) Critical processes can manifest sharp switching and/or sensitivity and can produce various steady, reproducible and/or reversible movements, but results depend on rates of processes during the critical moment.

In Quad Net operations, the plunge to zero, sharp switching and sensitivity depend on the rate of passage of the device through the critical moment, e.g., whether rapid or, in contrast, step-wise (quasi-statically). In other words, an important feature of critical processes in the Quad Net model is a variable rate of change that can either (1) resemble quasi-static processes of laboratory examples of empirical science and/or fluctuational processes of the Ising model; or (2) operate dynamically with a driving cause and a deadline, like the mechanical metaphor in the first step. Discussion here begins with quasi-static rates that are needed for the most sensitive elemental operations. Later it will be suggested that operations can be made more speedy and incorporate dynamical features by means of, e.g., long-range interconnections, momentum of moving body parts and material properties such as those resulting from training.

Limited to quasi-static operations, parallel features appear in domains of psychology and Quad Nets. In Quad Nets, quasi-static operations can pause during a critical moment and allow for an extended interplay of selective influences. These operations can occur in repetitive cycles, leading to production of "the same" result in each cycle — e.g., like repetitively picking the biggest onion in a bin or slicing a cake into equal pieces. Such results are reproducible. In scientific research into movements of human beings, a capacity to achieve "the same" result by means of different movements and despite perturbations is called "equifinality." In the examples, the performer's body may change postures between successive onion picks or cake slices while "the same" result is achieved in repetitive cycles of processes.

In dynamical cases, in contrast, a fast-changing driving cause may permit only a brief exposure to selective influences. When several selective influences vary widely, dynamical processes produce unique events — e.g., unique song performances, competitive games or civil trials. Such events are generally not reproducible. Training is possible, e.g., in a golfer practicing putting on a grassy green, who is attempting to employ a quasi-static selection process to produce a dynamical event that imparts a precise momentum to the golf ball.

Hence, an important parameter in Quad Net operations is the rate of the drive process, whether fast, or slow and steady, or small-step-by-small-step. A faster rate means less sensitivity and a greater likelihood of a failed performance. Conversely, overly-sensitive deliberated action can miss the deadline or can fail to produce the impulse needed to impart momentum. In repetitive processes, it is possible to vary the speed of the process and find the best compromise for a specific task. Training and practice can lead to a higher speed.

From another angle, the changing energy configuration in Quad Nets was suggested by metallurgical models of phase changes driven by changing "free energy" levels. In metallurgy, "free energy" specifies energy that drives conversions and transformations. (In the VE domain, there is "more than enough" energy; conversions and transformations are driven by device operations.)

Metallurgical models based on "free energy" describe changes in steel that occur when a red-hot form called austenite cools slowly into a form called pearlite or quickly into a form called martensite. In other words, a slow quasi-static cooling process produces one kind of steel called pearlite and a fast quench in ice water produces another kind of steel called martensite; and the two steels have different properties, e.g., as to appearance, hardness and brittleness.

Similar to processes in iron that produce pearlite, I suggest that critical processes in a QN assembly can be driven at a slow rate, producing ***steady movements*** that are suitable for weight-bearing movements of legs and arms such as walking up stairs or kneading a loaf of bread. Alternatively, critical processes can be driven at a fast rate, like those that produce martensite, producing sudden forceful movements, called ***saccadic***, that are sometimes needed by an organism in its struggle for life or in a fast, competitive ping pong game. In sum, a variable range of possible rates includes both steady movements (continuous, loaded, controlled) and also saccadic movements (sudden, forceful, ballistic).

Following models of physical phase changes, some processes that produce slow steady movements can be reversed; during a reversed process, reversal movements will be produced. After turning to the left, the head turns to the right.

In contrast, fast saccadic changes are generally ***irreversible***. In sum, I suggest that certain movements can be "taken back" but that others cannot; and that the difference depends, in part, on the rate of the process that produces the movement.

The Quad Net model therefore suggests a spectrum for movements with classes that are steady, controlled, reproducible and reversible at one end of the spectrum and classes that are saccadic, sudden, non-reproducible and irreversible at the other end. The location of a movement in the spectrum depends, at least in part, on the rate of the critical process that produces the movement.

A large-scale application of the spectrum principle distinguishes movements of hands and arms from those of legs and pelvis: manual movements have capacities for fine, careful movements at the steady/reproducible/reversible end of the spectrum and pelvic movements have capacities for sudden powerful movements at the saccadic/non-reproducible/irreversible end.

The pelvis and its vertebrae do have capacities for steady, fine and reversible movements, e.g., operating an automobile accelerator pedal or the pedals of an organ. Compared to the multitude of hand and arm movements that grasp, handle and operate objects, however, fine-movement capacities of the pelvis seem to be sparse and restricted. Similarly, sudden forceful movements of the arms and hands are relatively rare in the general population. Most of us can try to snatch at a mosquito but not much more. Some athletes like baseball pitchers and boxers develop capacities for sudden, forceful arm and hand movements but these are exceptional.

Eyes, fingers and tongue exemplify quick and highly variable movements. These body parts are small and light in weight and operate in low-friction anatomical structures. Movements of the eyes are easily reversible, a feature due to symmetries of a spherical eye and its muscular organization. Fingers have fewer reversible movements, e.g., adjusting a slider on an electronics instrument. Slow conscious movements of the tongue inside the mouth can be reversed, e.g., circling the lips, but not the quick unconscious movements of speech.

The division between reversible and irreversible movements has parallels in the psychological domain. Fast, saccadic, irreversible movements are discussed further below in connection with momentum. Reversible movements can be developed into what Piaget calls "systems of operations." In the following extract, from his *Play, Dreams and Imitation in Childhood* (1962) at 289, I would substitute "movements" for "objective transformations."

"A system of operations such as the elementary operations of arithmetic or geometry and logical seriations and nestings, can equally well be considered as a set of objective transformations successively reproduced through mental experience (imitative accommodation) or as a system of combinations resulting from the assimilating activity of the subject. Moreover, the characteristic feature of operations is their reversibility…"

(C) During a quasi-static critical selection process, a condition of indeterminacy arising during the critical moment may manifest a unique character that extends over the whole body and that leads to integrity of movement as the process progresses.

In all the physical critical selection processes, there is a central critical event (point or moment) with a character that is different from both the endpoints (the initial condition and the final condition). Conditions at the endpoints are not changeable but, during the critical event, change is occurring. I suggest that, during the critical event, there is an ***indeterminate condition*** that is intermediate between and different from both the initial condition and also the final condition.

Therefore, a critical selection process can be divided into two parts: first, a transformation from the initial condition into the indeterminate condition; and second, a transformation from the indeterminate condition into the final condition. In quasi-static processes, all initial conditions transform into the same indeterminate condition. In contrast, the transformation of the indeterminate condition into the final condition can require a selection between possibilities that is subject to influences.

The indeterminate condition is maintained by balancing, but balance is only barely maintained. Balance is maintained until it is lost during the critical event. Loss of balance constitutes the transformation of the intermediate condition into the final condition. There may be multiple possible ways to lose balance, but there is only one actual way in any particular cycle.

The indeterminate condition is called ***Shimmering Sensitivity*** in the Quad Net model. In a step-by-step or quasi-static process, it is possible to stop at the intermediate indeterminate condition and to maintain it for an extended period and thus to generate a sustained condition of Shimmering Sensitivity. This is a condition of undirected readiness. (Certain meditative practices have a similar effect in the psychological domain.)

One way to construct a condition of Shimmering Sensitivity is to start with the collective repertoire of all the possible final waves, next to construct small ***germinal fragments*** of each final wave and finally to locate germinal fragments in the quadnet field. Please see images above that contrast a shimmering condition with an extended wave. There are various ways to produce germinal fragments, e.g., a researcher sends signals over projections that are attached to strategic elements in a quadnet. See also reciprocating TQN's discussed below.

In a quasi-static process, as the quadnet enters the critical event, germinal fragments are generated and then compete amongst themselves. At the critical point, competition is just barely balanced and different kinds of fragments co-exist.

As the cycle proceeds and the conditions steps away from the critical point, some germinal wave fragments grow larger and other germinal wave fragments are extinguished. As the cycle period increases, germinal fragments connect with other fragments of their own kind and a growing body of one kind excludes all other kinds. The outcome of the competition can depend on small external influences or, as suggested below, on small variations in properties of material bodies of devices. It would appear that, at least for certain classes of input signals and certain processes, the quadnet produces a single final wave and achieves integrity of movement.

The QN construction has a parallel in psychological examples. Perhaps I am in a food store, standing in front of a rack of candy bars. I am inclined to eat a candy bar and will pick exactly one standard priced confection. My hand starts to reach for a gooey caramel but then pulls back at the thought of the goo. Next, my eyes catch the nut bar that I sometimes choose and I twist to view it directly. But wait, now I am stooping to pick a peppermint patty from the bottom shelf. I suggest that such impulsive preliminary movements resemble germinal fragments of waves in the Quad Net model that finally coalesce into an actual movement.

Recall the metallurgical processes where one material structure turns into another structure and the transformation is modeled by changing "free energy" levels. Conditions are stable when free energies of two structures are different — the structure with the lower free energy is the actual structure — but conditions become unstable when the free energies of the two structures are equal. When free energies are equal, structures change easily into each other.

Likewise, the polarity of a cluster of elements in the Ising model is easily changed at the critical point. See examples in Fig. 5, above.

Similar to metallurgical models and the Ising model, in the TQN, during a critical moment of Shimmering Sensitivity, germinal fragments of waves each have a small energy content and can be easily changed into each other by small influences. Germinal wave fragments that combine into a larger fragment have more stability and greater capacity for growth than fragments that remain isolated; and the largest fragment will tend towards an integrated movement of the whole body as the cycle proceeds.

Metallurgical processes of ***annealing*** are also suggestive. During a martensitic phase change in steel, material undergoes a rather violent transformation, resulting in a structure that has a multitude of deviations from a perfect structure, called ***dislocations*** that results in internal strains and tiny cracks. Dislocations tend to make a metal both hard and brittle, similar to window glass. Suppose that you take a piece of steel with many dislocations and load it cyclically with an external force, e.g., in a beam under a floor that flexes as people walk on the floor. Over time, movements generate new dislocations and cause dislocations to migrate and aggregate at fixed points. A bar with many aggregated dislocations will fracture.

To avoid fracture, it is possible to re-heat the piece of steel, without a load, to a temperature that is close to but below the temperature of the phase change. At such a temperature, nuclei are wobbly in positions and shift around so that internal stresses are able to relieve themselves spontaneously. Dislocations become mobile and are dispersed, even canceling each other, leading to more stress relief. When the steel is cooled, it is stronger and less subject to fracture. This process is called annealing.

In the Quad Net domain, a similar process of "down, then up, then down again" is called ***phasic cycling***. Each cycle results in a more uniform pattern. Cycles can be maintained and controlled through pulse patterns circulating in identical devices. The process has a parallel in a deliberative decision in the psychological domain.

## 3. Critique of the primal model; layers of further development

The primal QN model — based on a quasi-static critical selection process in a Toroidal Quad Net (TQN) —has important restrictions, limitations and uncertainties. Operations of the primal model are slow, requiring an extended period of indeterminacy during a prolonged critical moment. Signals move step-by-step across a field of elemental devices. The repertoire of final movements is very small. Whether integrity of movement can be achieved, and under what circumstances, is difficult to determine.

To overcome these shortcomings, proposed developments take the form of layers. First, the primal model is augmented by adding long-range interconnections and operations involving two TQNs and larger assemblies. Second, the model is extended to incorporate dynamical principles of momentum and saccadic (jerky) movements. Third, a material body has its own integrity that is modeled by means of whole-body material properties of elasticity, entrainment and training.

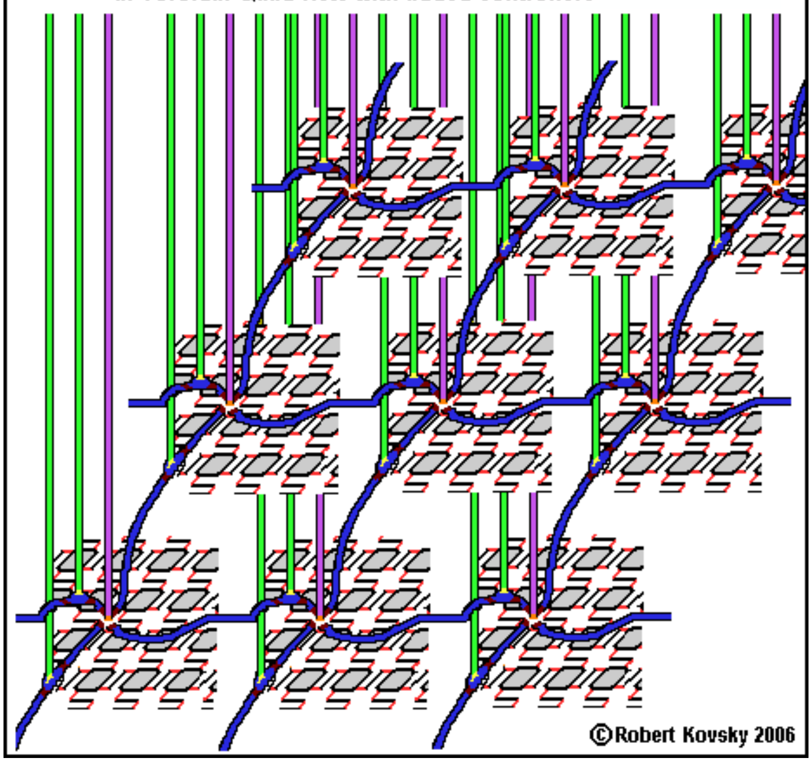
Based on these layers of development, I suggest that an exercise of freedom manifesting integrity of movement has components of rationality, action and material integrity. Here, components are separately discussed. In modeling actual movements, the combination of components at any moment would depend on the task and the circumstances. Each component contributes to integrity of movement in a different way.

### a. Long-range interconnections augment the primal model.

The first layer of development augments original short-range nearest-neighbor connections in a Primal Quad Net and TQN with new long-range projections. One way to do this is to start with multiple TQN's and interconnect them using long-range projections. In the plenary Phase Transfer Controller (PTC) constructed here, transfers of wave phases between TQN's are speedy and preserve integrity.

Image 30 Phase Transfer Controller, elemental details

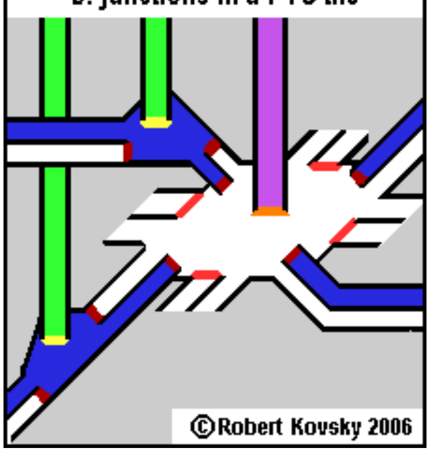
a. Toroidal Quad Nets with added controllers




A representative set of connections is shown in the adjacent Image 30a. Gray quadnets are interconnected by blue elemental devices that each have two pairs of long-range projections. Image 30b shows an expanded view of one set of interconnections.

In a plenary design, every elemental device in every quadnet has four short-range and four long-range interconnections. Long-range interconnections are based in blue intermediary elemental devices.

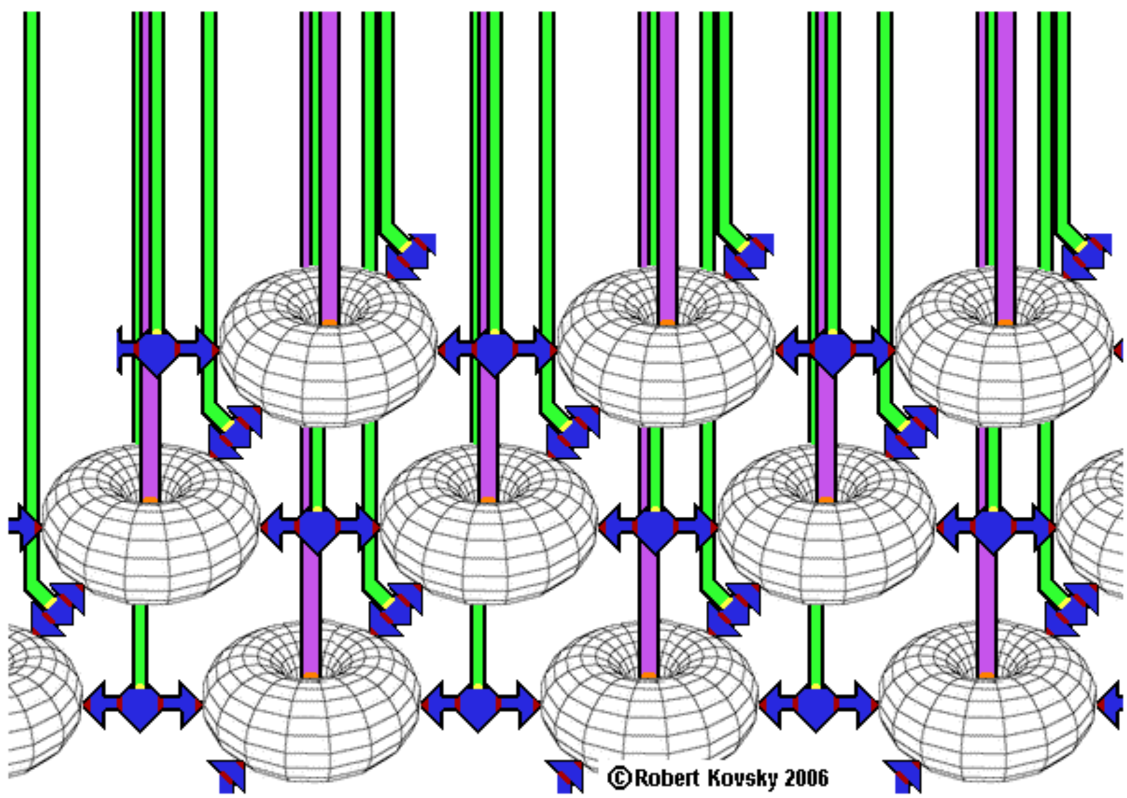
b. junctions in a PTC tile




Each elemental device in the PTC, whether in a QN or interconnections, is controlled by a vertical projection that comes from a remote location (eventually another quadnet). Vertical projections, called "controllers" in the PTC, carry the driving signals that initiate and govern the cycle of operations of an elemental device. First, purple controllers start the critical selection process in the QN; when the final wave is established, green controllers transfer that particular wave to another TQN, which is passing through a critical moment.

The overall architecture and functional organization of the PTC is shown in Image 31 below.  Every pair of neighboring toroidal quadnets is connected by an ***interconnection cable*** of elemental devices collected into a double-headed blue arrow.  Each TQN has n×n elemental devices and each interconnection cable also has n×n elemental devices.  Controllers are also cabled, with n×n projections in each controller cable.  However, if cabled operations are restricted to a synchronous mode where all elemental devices in a TQN or interconnection cable participate in a single common process, a controller can be reduced to a single long projection from the remote location, along with a ramified distribution network close to the elemental devices.

**Image 31   Phase Transfer Controller, architecture and functional organization**




The plenary PTC resembles a quadnet, except that a quadnet is made up of elemental devices and the PTC is made up of quadnets.

As the simplest example of a small assembly, a phase transfer between two connected TQN's is shown below. It resembles a "clear and add" instruction in assembly language for computers. In Image 34, the transferor TQN on the left has a fixed phase and the transferee TQN on the right goes through changes: first silencing a pre-existing phase and then becoming ready to receive the new phase over the interconnection cable. The CQN between the two TQN's used in this design has a single discharge window for all elemental devices in the cable.

Image 33 shows changes in a controlling quantity ε that tracks signals over controller cables to the transferee TQN and to the interconnection CQN. Three instants — a, b and c — cross-reference conditions in Images 33 and 34. Between instant a and instant b, the change in the value of ε in the transferee TQN silences that TQN. Just after instant b, the change in the value of ε in the CQN activates the transfer.

Image 33: ε's during phasic transfer

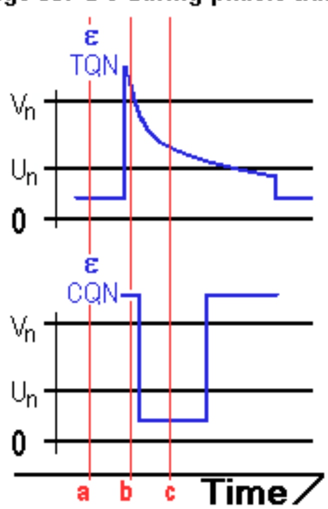



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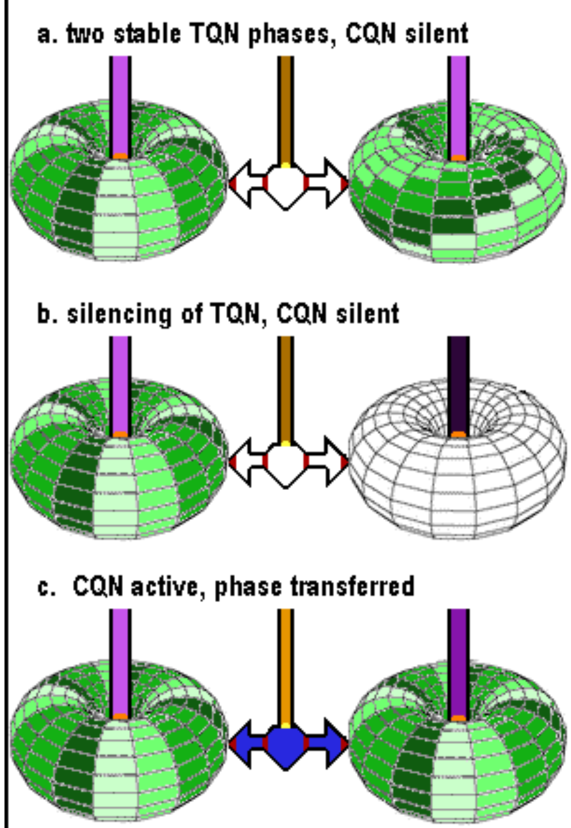
Image 34: Controlled phasic transfer

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As discussed above, it is possible to vary the number of elemental devices in interconnection cables. The time periods required for operations can also varied. In the full plenary system of the PTC, a phase can be transferred via a quick operation, with a transfer period of less than 10δ, where δ is the responding period of an elemental device. Similar results might occur if the number of elemental devices in an interconnection cable is uniformly reduced to one-fourth of the number of a plenary set and the transfer period is doubled. In another reduced system of interconnections, a “cluster” of elemental devices in a TQN would be interconnected to “clusters” in other TQN’s via one-to-one correspondences; and elemental devices surrounding clusters would lack interconnections to other TQN’s. Signals from clusters would spread the selected signal to surrounding elemental devices. A small cluster can control an entire TQN if other clusters are silent and enough time is available.

A further development introduces a reciprocating arrangement that also “mixes” the phases, providing for a quicker and more thorough passage through the critical moment. Instead of a connection cable that establishes one-to-one correspondences, a cluster of elements in one TQN is projected onto multiple dispersed clusters in the other TQN. And reciprocally from the other TQN to the first TQN. In reciprocating operations, a slight preponderance of one phase at the critical moment will quickly evolve towards complete occupation of the TQN’s.

b. Addition of a momentum principle extends the model into dynamical domains

In the second layer of development of the QN model, momenta of moving body parts drive certain elemental operations. In other words, momentum-driven dynamical operations are developed here as extensions of quasi-static operations. This development is rather limited. In fish and birds, it appears that sudden momentum-driven movements are general and common and that quasi-static movements are more specialized and specific. Another project might begin with sudden movements and develop lower activations as special cases.

The first extension from quasi-static conditions is to "steady" movements where uniform momentum is sustained against friction, such as movements of stirring a pot, walking and rowing. Steady movements can be steered by means of cyclical adjustments.

In a further extension, more sudden dynamical operations produce movements that correspond to "saccadic" movements in actual life, which are movements that impart momentum, such as jerking, tossing, throwing, punching, hopping and kicking.

The importance of momentum in exercises of freedom is illustrated by the bicycle. It is easy to balance on a moving bicycle but difficult to balance on a bicycle that is stationary. Reasons for the difference are shown by tricycles and by "training wheels" for children that are attached to the rear wheel of a bicycle; such vehicles are self-balancing even when stationary. A child can learn to acquire and lose momentum on self-balancing vehicles but additional skills are necessary to balance on a bicycle.

Suppose that a skilled person is standing on and over a bicycle, with one foot on a pedal and with the other foot on the ground. The person is leaning on the grounded foot; and the hands holding the handlebars keep the bicycle in a fixed position.

To get the bicycle into motion, a preparatory movement is required that brings the foot on the pedal into its highest position, where the leg is ready for a powerful down-thrust. Then, being ready, the person performs a saccadic movement that begins with pushing up from the ground with the grounded foot, detaching the foot and getting the body into a more upright position — and that proceeds with a sudden down-thrust on the pedal by the ready foot. That thrust fortifies the push that has been getting the body into an upright position and completes that movement, even going over to the other side. Meanwhile the once-grounded foot finds its pedal and contributes its own down-thrust, less sudden and forceful, that restores balance to the body in an upright position and that leads to smooth motion where alternating pedal thrusts maintain balance.

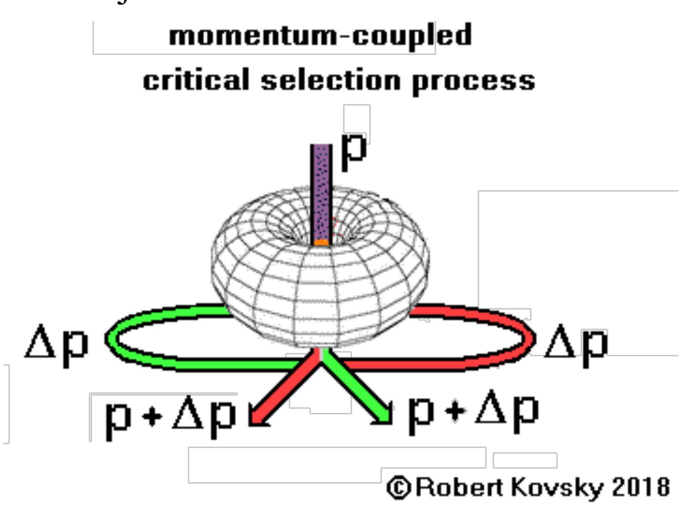
While the bicycle is moving steadily with a constant momentum, additional muscular movements of the arms can steer it in exercises of freedom. Birds in flight have similar capacities for free movement but with a more restricted repertoire since they must use their wings for both propulsion and steering.

In the foregoing sequence of movements, an initial saccadic movement starts the motion, which then evolves. The starting movement is the most powerful and subsequent movements are of lesser strength. Finally the course of movements reaches a steady condition.

Training wheels enable a child to learn to produce the powerful leg thrusts needed to propel the vehicle. The child's body must re-balance during each stroke and this re-balancing requires practice. The self-balancing vehicle provides a platform for this exercise of freedom. Then, further learning and practice are required when the training wheels come off.

A distinctive sequence of movements is performed by skilled bicyclists waiting at traffic lights and stop signs. Some do not want to dismount but cannot balance while the bicycle is stationary. So they jerk back and forth in short saccadic movements and rebalance by means of the handlebars during each movement. There are alternating jerks and rebalances, which correspond to the driving cause and the selective cause in the dynamical system of the first preparatory step. Here, the rate of jerks sets the tempo and selective causes have to fit into the deadline set by the jerks. The adjustment to balancing via the handlebars is also timed to occur at the maximum forward momentum and angular momentum of the forward wheel. As a result, in the skilled and practiced cyclist, momenta of movements drive selection processes and the adjustments are adjustments to momenta.

The adjacent figure suggests a TQN model of saccadic balancing or steering. Signals driving the controller are based on the momentum **p** of a body part during the prior muscular stroke. Other signals influence the selection process and the variable product of the process Δ**p** is combined with the original **p** to produce the next muscular stroke with a momentum **p** + Δ**p**.



c. integrity of material media and principles of elasticity, entrainments and training suggest further developments

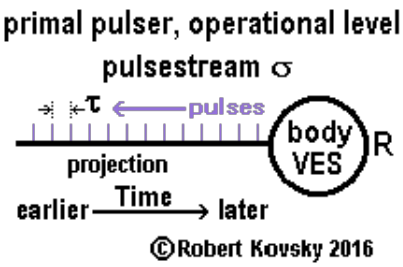
Principles of material integrity round out the Quad Net model of freedom. Obviously, the capacity of an animal to perform integrated movements depends on the integrity of its body. In the human body, movements of bones and muscles are united and held together by connective tissues (ligaments, tendons, bursa) and by the collective weight and structure of surrounding organs, fascia and skin. On the microscopic scale, each cell has its own material integrity and its own muscular activity; every cell interacts directly with neighboring cells and indirectly with the external world. A human body is made up of trillions of living, interacting, moving parts.

In the third layer of the QN model, whole-body material properties are added to an embodied collective structure of elemental devices, which is called a ***module***. Modules have whole-body material properties, e.g., properties of elasticity, entrainment and training — resulting in classes of modular bodies with variable properties. The aim is to model integrated movements of animals that require training and practice, during which movement patterns become embodied in joints, nerves and muscles.

(1) Following Hooke's law, scientists have stated and developed principles of ***elasticity*** in many quasi-static applications, e.g., metal springs, rubber bands and mattresses. I have included principles of elasticity in Wavemaker designs and in A Tube for Transport in the *Paradigms* project. In possible further developments, quasi-static applications of Hooke's Law can be extended into dynamical movements. A limited but useful dissipation principle is readily grafted on.

(2) If identical mechanical clocks are placed in a row on a wooden table, their tickings will adjust so that all the clocks tick at the same instant or synchronously. This phenomena is called ***entrainment***. It is clear that vibrations in the table are involved in entrainment. If several identical clocks, each in a sealed capsule were brought close to each other in a vacuum with no material connection, presumably each would continue to tick independently or asynchronously.

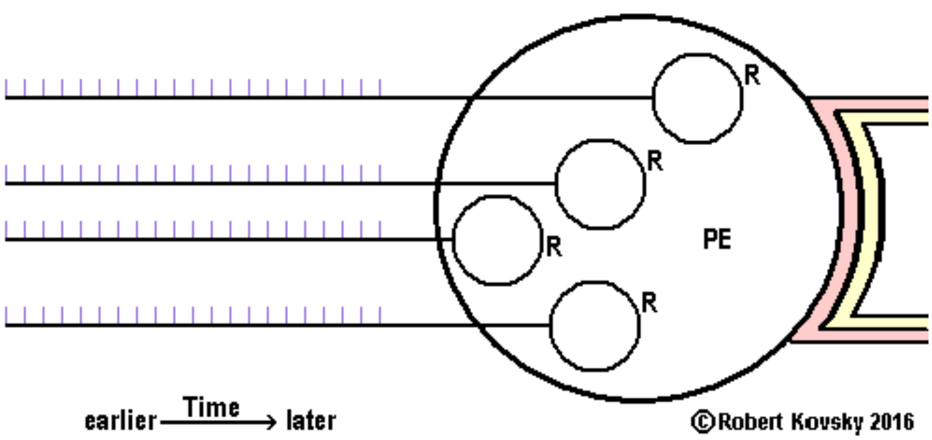
Principles of entrainment are illustrated by pulser designs discussed in detail in the *Paradigms* project. The image below shows the design of a primal pulser, the simplest Virtual Energy (VE) device. VE flows into the pulser at the rate R and is stored in a Virtual Energy Store or VES in the body of the device.




When the quantity of VE in the VES equals a certain amount, called one bang (!), that ! of VE is discharged as a pulse on the projection, emptying the VES. The period between successive pulses is $\tau$. The operational definition of the device is $R \times \tau = !$. A steady inflow of VE, namely R, is converted into a series of instantaneous pulses, the pulsestream $\sigma$.

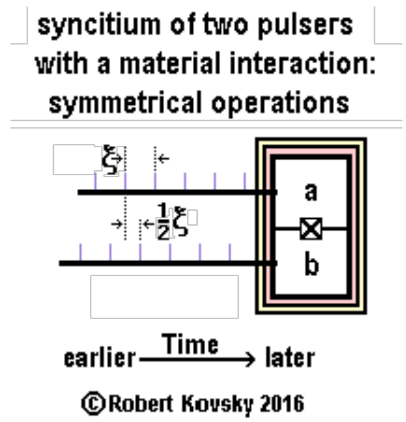
In the design for conglomerated pulsers below, four pulser devices are contained or conglomerated in a module that provides a paradigmatic environment (PE). Virtual Energy (VE) is supplied from an external source, passes through the PE and is stored in each pulser at a rate R. The PE also stores VE and serves as an exchange medium for transfers of VE between pulsers. A discharging pulser discharges VE into the PE as well as over a projection. The principle of entrainment states that VE is distributed in the PE so that all pulsers discharge synchronously. The collective conglomerate participates in such whole-body discharges.

entrained pulsations of conglomerated pulsers



Another variation is shown in the pulser design below. Two pulsers, a and b, are neighbors in an arrangement, called a ***syncytium***. The syncytium is a very simple means for assembling pulsers that can be developed into a string, a sheet or a solid.

Each pulser has its own R source of VE but they also exchange VE through a junction between the pulsers, denoted by a box. When pulser a discharges, VE passes through the junction, combining in the VES of b, along with an inflow R from the surrounding source. Passage of VE through the junction is delayed. The two pulse patterns alternate in a symmetrical time pattern. The symmetrical production of pulses is stable if a perturbation is introduced.




Examples of the material principle of ***training*** are provided by leather and other materials in which permanent creases and folds can be worn in. Metaphorically, training resembles the results of flowing waters that wear grooves, gullies and canyons in geographical terrain. In the Virtual Energy model, flowing VE wears channels in physical materials in device and modules. In the psychological domain, I suggest that a "trained individual" has developed a network of channels in material bodies for pulsing energy flows that are ramified into branches, where each branch drives a possible movement. Flows of energy are diverted at branch points to suit the distribution needs of the operator.

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The foregoing constructions present the Quad Net model of freedom in its present stage of development. Chiefly, there are three components, the rational component based on critical selection processes, the component of momentum and the component of material properties. Most of the attention has been focused on the rational component. This emphasis is a result of a means having become available for rational constructions, namely, quadnets. Quadnets also highlight shortcomings in understanding muscular movements of animals.

But first, whatever its shortcomings, the Quad Net model can be applied to describe the ping pong stroke previously discussed. Recall that ping pong strokes in a volley are cyclical movements with a lot of possible variations between cycles. Suppose that we enter the cycle while the ball is in flight just after the protagonist player has finished producing a stroke and before the opponent has produced a stroke. The player's body is activated and stationary in a ready position where it can move to a range of positions where the next stroke will be produced. Muscular movements of the player's eyes follow movements of the opponent's body. When the opponent produces a stroke, movements of the player's eyes become transformed into triggers for the first, saccadic movement of the player's body. That movement is based in the pelvis and has a speed and direction roughly selected to position the player's body for the return stroke. This movement resembles a reflexive movement and is charged with a high initial momentum.

In the second stage of the stroke, most of the momentum is dissipated as the player's body reaches the stroking position but substantial momentum is channeled into the paddle arm. The channeling is influenced by delayed visual images of the opponent's paddle, by tracking of the ball with eye muscles, and by momentary thoughts and feelings of the player such as intentions. The particular stroke (flip, loop, drive) is selected.

Finally, in the third stage, the actual stroke is performed. It is in this stage that channeling of momentum and selectional processes are most strongly influenced by training and practice. I suggest that the final precision movements of a ping pong stroke depend chiefly on material properties.

## 4. Contrasts between the Quad Net model and consensus science

Application of the Quad Net model to ping pong provides a perspective on exercises of freedom. In brief, I suggest that muscular movements in ping pong are products of animal bodies that are guided by images. It is an animal body that produces the integrated ping pong stroke, along with the intentions and the images of the objects. Although we understand that the body produces the stroke, we do not understand how it is done. Understanding how bodies produce images is even more difficult.

Our inability to understand integrated movements and images produced by animal bodies clashes with claims of leading scientists. These are claims of "platonic science" discussed in my essay, *How to solve "free will" puzzles and overcome limitations of platonic science* (2016).

The essay includes a critique of the science of physics as that science is described by Richard P. Feynman in his famous *Lectures on Physics*, which were given to entering students at the California Institute of Technology. According to Feynman, "everything is made of atoms," all material properties are consequences of "quantum mechanics" and everything in the universe obeys "Laws of Physics." One of the most important "Laws of Physics" is Conservation of Energy, that is declared to be universal, eternal, exact and perfect.

In another presentation, *The Character of Physical Law* (1965) at 151, Feynman provides a list of "elementary particles":

> With these particles that I have listed, all of the low energy phenomena, in fact all ordinary phenomena that happen everywhere in the Universe, so far as we know, can be explained. … For example, life itself is supposedly understandable in principle from the movements of atoms, and those atoms are made out of neutrons, protons and electrons. I must immediately say that when we state that we understand it in principle, we only mean that we think that, if we could figure everything out, we would find that there is nothing new in physics which needs to be discovered in order to understand the phenomena of life. … In fact, I can say that in the range of phenomena today, so far as I know there are no phenomena that we are sure cannot be explained this way, or even that there is deep mystery about.

Another example of such claims is set forth by computer intelligence advocate Marvin Minsky (*Society of Mind*, § 30.6):

> According to the modern scientific view, there is simply no room at all for "freedom of the human will." Everything that happens in our universe is either completely determined by what's already happened in the past or else depends, in part, on random chance. Everything, including that which happens in our brains, depends on these and only these:
>
> *A set of fixed, deterministic laws.* *A purely random set of accidents*
>
> There is no room on either side for any third alternative. Whatever actions we may 'choose,' they cannot make the slightest change in what might otherwise have been – *because those rigid, natural laws already caused the states of mind that caused us to decide that way.* And if that choice was in part made by chance – it still leaves nothing for us to decide.

I prefer the view expressed by Clifford A. Truesdell in Truesdell & Noll, *The Non-Linear Field Theories of Mechanics* (1965) at 1:

> "Matter is commonly found in the form of materials. Analytical mechanics turned its back upon this fact, creating the centrally useful but abstract concepts of the mass point and the rigid body, in which matter manifests itself only through its inertia, independent of its constitution; 'modern' physics likewise turns its back, since it concerns solely the small particles of matter, declining to face the problem of how a specimen made up of small particles of matter will behave in the typical circumstances in which we meet it. Materials, however, continue to furnish the masses of matter we see and use from day to day: air, water, earth, flesh, wood, stone, steel, concrete, glass, rubber..."

Isaac Newton also entertained views different from those of consensus science, writing to a colleague:

> God who gave Animals self motion beyond our understanding is without doubt able to implant other principles of motion in bodies which we may understand as little. Some would readily grant this may be a Spiritual one; yet a mechanical one might be showne… (reported in J. Gleick, *Isaac Newton* (2003) at 105-106.)

In my view, consensus science is a rigid and closed-minded discipline based on geometrical constructions in empty space and mechanically-connected movements that occur in detached time. Constructions in detached time include planetary orbits and adiabatic and isothermal processes used in conserved energy thermodynamics. There are some material bodies for which such descriptions are suitable, but not living animal bodies.

Previous discussions in this project provide specific examples of the shortcomings of consensus science. There is nothing in atomic theory or quantum mechanics to explain the startling resemblances in traces for latent heat in water/steam and saturation magnetization in magnetite. Such theories provide no suggestion of how traces of multiple materials "collapse" into a single trace through scaling. Although physicists borrowed "quantum mechanical" principles called "renormalization" to explore the critical point, the applications are opportunistic and quite different from the original application. Feynman, an inventor of renormalization, acknowledges its shortcomings and calls the original application "a dippy process." (See my *Patchwork of Limits* article on limitations of science.)

Water is the most abundant material in actual life. But water eludes the grasp of consensus science. As stated in my free-will puzzles essay:

> "Seventy three anomalous properties of water" are described in an encyclopedic website on the science of water (Chaplin) that states in the Introduction: "Water is the most studied material on Earth but it is remarkable to find that the science behind its behavior and function are so poorly understood (or even ignored), not only by people in general, but also by scientists working with it every day. It can be extremely slippery and extremely sticky at the same time; and this 'stick/slip' behavior is how we recognize the feel of water. The small size of its molecule belies the complexity of its actions and its singular capabilities. Many attempts to model water as a simple substance have failed and still are failing. Liquid water's unique properties and chameleonic nature seem to fit ideally into the requirements for life as can no other molecule."

> [M. Chaplin, Water Structure and Science, http://www1.lsbu.ac.uk/water/index.html]
>
> Other researchers state:
>
> It may appear as a paradox that we should be able to describe our universe at extremely large scales, and also at the scale of elementary particles, but yet [are] unable to understand the liquid that we are made of.
>
> [B. Cabane, R. Vuilleumier, The physics of liquid water, https://hal.archives-ouvertes.fr/hal-00015954/document]

In the free-will puzzles essay, I discussed failures of consensus science to explain formation of snowflakes and turbulence in fast-flowing water. Another poorly understood aspect of water is how it affects movements on the microscopic scale. A concept based on flows of water in the laboratory, the ***Reynolds number***, gives some guidance. "Reynolds number represents a ratio of inertial force to viscous force." (Brusca & Brusca, *Invertebrates* (1990) at 50.) When large animal bodies, e.g., humans or ducks, move through or on water, inertial force predominates and the body continues to move or glide while resting a bit; movements are characterized by a "high Reynolds number."

In contrast, movements of one-celled animals are characterized by "low Reynolds number." "But viscosity becomes important, increasingly so, as body size and velocity decreases (i.e., as the Reynolds number decreases.) Small animals swimming through water have been likened to a human swimming through liquid tar, or thick molasses." (*Id*.)

In my approach, I attribute puzzling, incomprehensible and paradoxical phenomena of life to properties of material bodies, including human bodies. I suggest that our minds are very good at problems involving space and geometry so we try to use such spatial methods to describe properties of material bodies. We discover and exploit materials that have properties that conform to spatial methods, such as materials with "linear" properties that obey Ohm's Law for electrical resistance or that come close to the requirements of the Ideal Gas. "Homogeneous materials" and "quasi-static processes" also conform to spatial methods. Consensus science neglects to notice shortcomings in spatial methods, but presumptively extends them to comprehend "the universe."

I suggest that we acknowledge limitations and defects in our mental concepts. We do not understand itching and scratching. Our minds have not solved simple problems of animal locomotion. Nor does a solution appear likely. Simple problems include movements of one-celled animals or protozoa, such as amebae and paramecia. Why and how does an ameba extend a pseudopod in one direction rather than other directions? "Although biologists have been studying ameboid locomotion for over 100 years, the precise mechanism is not yet fully understood. The physiological basis of ameboid movement is probably essentially the same as that of vertebrate muscle contraction, involving actin, myosin, and ATP." (Brusca, 51.)

Protozoans called ***ciliates***, e.g., paramecia, possess active filaments or ***cilia***, that move the organism and generate feeding currents. Cilia are densely distributed over the body of the organism; "patterns of ciliation on the body [of a paramecium] are extremely varied and this allow a range of diverse locomotor strategies … The beating of the ciliary field occurs in metachronal waves that pass over the body surface []. The coordination of these waves is apparently due largely to hydrodynamic effects generated as each cilium moves. Microdisturbances created in the water by the action of one cilium stimulate movement in the neighboring cilium, and so on over the cell surface." (Brusca 150-152.)

According to principles of low Reynolds number, movements of a paramecium occur in a thick viscous medium. The cilia pull the organism through the medium; but it also appears (as stated by Brusca) that timings of pulls are triggered by movements of the medium. It appears that movement involves body waves and waves in the medium. Or, perhaps, there are two coordinated waves, one inside the organism and the other outside.

Here again, methods of consensus science, based on spatial theories and computations, are blocked by apparently insurmountable problems. As an alternative to spatial methods, I construct new methods based in time. As an alternative to theories. I propose designs for devices. Elemental designs for devices depend only on an axiomatic math-like Virtual Energy Store (VES) and operations with respect to the VES. Then, in additional layers of development, devices also depend on material properties and physical momenta.