A PATCHWORK OF LIMITS:

Physics Viewed From an Indirect Approach

The direct approach to physics assumes that reality is inherently and entirely structured and that the structures can be described by mathematics. The indirect approach takes a contrasting position: that reality is not inherently structured, but partially structurable through human activity and cognitive processes; and that mathematically-based structures are projected onto, or built into, selected aspects of reality. The indirect approach includes a psychological model of structuration and a method for evaluating the success of structural representation. Success appears perfect in one area of physics, relativity theory; but in others, exemplified by thermal physics dealing both with ideal gases and also with systems in the critical state, structural representations appear successful only at limit points; moreover, the concepts employed at the limit points clash irreconcilably. These conclusions challenge widely held beliefs in the comprehensive explanatory power of "laws of physics" and "emergent behaviors."

See you now —
Your bait of falsehood takes this carp of truth;
And thus do we of wisdom and of reach,
With windlasses and with assays of bias,
By indirections find directions out.
Hamlet, Act II, Scene i.

...nature is never spent;
There lives the dearest freshness deep down things.
Gerard Manley Hopkins

Part I General Principles

§1 Introduction

There is a widely held belief that mathematical formulations ("laws of physics") can comprehend physical reality. For example Penrose (1989, 433) writes presumptively about "the mathematical scheme which governs the structure of the universe." The belief in comprehensive mathematical formulations implies belief in exact correspondences between mental concepts and structures inherent in reality. These beliefs, the "***direct approach,***" has serious problems, including an inability to account for the history of conceptual changes in physical theories, the existence of multiple, divergent theories bearing on a single subject and the contrasting features found in such theories. Nonetheless, the direct approach is assumed by practicing physicists as a matter of course and exerts a compelling attraction on all reared in the scientific tradition.

In this essay I pursue an alternative ***indirect approach*** that begins with principles so contrary they may at first appear indefensible. If, however, they are followed through, they will circle round to resolve problems with the direct approach and to illuminate additional questions that have long been sources of perplexity. The chief advantage of the indirect approach is that it is possible to retreat from the contrary principles when retreat is required, allowing for a mixture of the contrary principles and partial negations thereof. A partial negation of contrary principles corresponds to a ***part*** of the view seen from the direct approach, but not the whole. The indirect approach allows for partial retreat, but the direct approach does not allow for any retreat at all.

It will be seen that a mixture of direct and contrary principles can be shaped so as to account for those parts of reality that are not amenable to the direct approach. For example, it accounts for the presence in physical theories both of concepts that appear directly referential to reality and those that do not, and it provides a qualitative measure of "approximate truth." (Boyd 1990)

Seen from the indirect approach, some physical theories, e.g. relativity, appear to correspond exactly to aspects of reality (or indistinguishably so). Others, e.g. thermal physics, appear to correspond to aspects of reality only in the sense of a limit, as conditions under which the theories are tested approach ideals of isolation, simplicity and constraint. Theories do not always fit neatly together. Taken as a whole, I conclude that physics is a "patchwork of limits."

§ 2 Overview of the Indirect Approach

a. A contrary ontology

The direct approach assumes that reality is inherently structured. The indirect approach begins with the contrary assumption, that reality is not inherently structured. Rather, structures are products of cognitive processes and projected onto reality both in perception and through action. "Structures of experience" is the foundational concept. Structures of experience include mathematical structures.

According to the indirect approach, we test structures of experience against reality and discover aspects of reality, "domains," that appear to be described by such structures, at least partially. A domain is defined by the structures of experience that appear descriptive and by the scope of their application. We may conclude that some domains are exactly described by such structures. It is within the potential reach of the indirect approach that all of reality might be so described and the indirect approach would then completely reduce to the direct approach. Such a complete reduction is not necessary and, indeed, analysis of ***actual theories of physics***, as below, leads to the conclusion that it cannot be reached.

b. Epistemic heterogeneity

The direct approach assumes that there is an inherent structural substrate common to both reality and concepts in physics, in the form of mathematics. I call this assumption a belief in epistemic ***homogeneity***. Viewed from the indirect approach, reality and experience are of different kinds: experience is structured but reality is not entirely structurable, giving rise to epistemic ***heterogeneity***.

Epistemic homogeneity leads immediately to a belief in valid knowledge but it creates problems in accounting for error and illusion. Error and illusion are inherent in heterogeneity, but can be reduced through development.

A significant reversal concerns the problem of defining the boundary between valid knowledge and speculation or metaphysics, what Popper (1959, 34-39) called "the problem of demarcation." For homogeneity, valid knowledge is ***inside*** the boundary, and the approach to the demarcation problem is chiefly through logic. See, e.g., Wittgenstein (1918).[1]

Viewed from the indirect approach, it is speculation and metaphysics that are at the center and valid knowledge is directed ***outwards*** (with exact validity on a perhaps-reachable horizon). That is, all structures share a common origin in invention and are

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[1] In 1930, Wittgenstein "gave up the attempt to repair the structure of the *Tractatus*, and abandoned altogether the idea that here ***had*** to be a commonality of structure between the world and language." (Monk, 1990, 274) (emphasis in original)

displaced from the origin to the extent they are proved ***in fact*** to refer to matters in reality. Objectivity is constructed, rather than a given.

c. Development, Multiplicity and Approximation

The indirect approach accounts in a natural way for the history of development in scientific theories, for multiple theories bearing on a single subject matter and for approximate theories. Structures are invented and tested against reality; reality is divided into domains according to the structures that are applicable thereto. Ingenuity can invent structures that are applicable to broader aspects of reality or that constitute closer approximations to narrower aspects. Different structures are invented in the course of development, but they build on earlier inventions.

Thus, a student of physics learns theories roughly in the order they were developed. E.g., Newtonian physics is a prerequisite to relativity theory or quantum mechanics and principles of electromagnetism, starting with Ohm's inverse square law of electrostatics and progressing through Maxwell's dynamical equations and beyond, are taught more or less in historical order even when modern mathematics is used throughout.

Matters in reality that are not entirely structurable are sometimes approximated by multiple, different theories. For example, no theory comprehends the electrodynamics of moving media, but there are several forms of approximation. “We shall see four examples of formulations that ... differ from one another in the particular field variables used, in the meaning of the field variables, in the form of Maxwell’s equations and the constitutive laws, and in the forms and numerical values of the various energies, co-energies, powers, momenta, and potentials. In fact they differ in just about every aspect except the numerical values of predictions of experimental measurements." (Penfield & Haus 1967, 4) (See also § 4, example 3 below)

d. Incongruence

Both the direct and the indirect approach assume that reality is integral, i.e. that it consists of a single piece. Epistemic homogeneity would lead to an integral system of scientific theories. Different theories might be applicable to different aspects of reality, but they should fit together directly, like pieces of a jigsaw puzzle. I say that such theories are ***congruent*** with one another.

Epistemic heterogeneity is consistent with the existence of theories that do not fit together directly. I call such theories ***incongruent***. Points of mismatch are called ***incongruities***.

The argument may be stated in the form of a "syllogism."

Reality is integral.

Mathematical structures can correspond exactly to and comprehend reality.

∴ Mathematical structures corresponding to reality form an integral system.

The contrapositive of the "syllogism" states that, for an integral reality, incongruities imply a failure of exact correspondence, consistent with heterogeneity.

Congruence and incongruence characterize relationships involving matters within experience, e.g. theories of physics, in contrast to homogeneity and heterogeneity, which would characterize the relationship between experience and a reality outside experience. Congruence and incongruence are, therefore, subject to direct examination, while the underlying homogeneity/heterogeneity issue is not. Hence, examination of theories of physics for congruence and/or incongruence can support implications about homogeneity and/or heterogeneity.

Those who follow a direct approach seek to deny the existence of incongruence and to smooth over difficulties in harmonizing theories, such as difficulties encountered in connection with the EPR paradox and the Aharonov-Bohm Effect. In following an indirect approach, such matters are subjects for analysis, not for attempts at resolution.

§3 Processes of structuration

According to the indirect approach, structures are generated by cognitive processes, including action. The indirect approach therefore calls for a psychological model for the generation of structure. A full presentation of my model is beyond the scope of this essay, but discussion of basic features is appropriate. The basic model is crude, but can be developed for greater refinement and scope of application.

The inspiration for my model is the work of Piaget, who pioneered the psychology of the development of intelligence in children. (See especially Piaget 1937, *The Construction of Reality in the Child*.) Piaget was temperamentally committed to a homogeneous viewpoint, but, "even after close scrutiny, the reality of cognitive structures remains unclear. Sometimes, Piaget gives the impression that he believes in the ontological reality of these structures. Sometimes they seem simply abstract models of behavior." (Gruber & Vonèche 1995, 742) Piaget concluded that there is a "fundamental process of intellectual creation, which is found at all levels of cognition, from those of earliest childhood to those culminating in the most remarkable of scientific inventions."

(Piaget 1970 at 78; see also Piaget 1937, 356, 370 and Kuhn 1971) As stated in Gruber & Vonèche (1995, xxxviii), "The function of cognitive growth is not to produce schemes that are more and more veridical copies of reality, but to produce more and more powerful logical structures that permit the individual to act upon the world in more flexible and complex ways."

In my model, the product of structuration processes is an element of structure, called the ***context-detail pair***. A detail appears within a context; a context contains details. For example, the relationship between a set and a member thereof is a context-detail pair. Another example is the relationship between a person's face and his or her nose; set theory cannot apply here because neither face nor nose is sharply defined. In its crudest form, the context-detail pair corresponds to Piaget's relation between whole and part. (Inhelder & Piaget 1969, 8)

Relational psychological processes can move in two reciprocal directions. (Piaget & Inhelder 1969) Beginning with a context, generate a detail thereof: a process called *explication*. Or, beginning with a detail, generate a context that contains the detail: a process called *implication*.

The two processes work together (co-ordinate) to isolate details, to construct conceptual contexts that contain the details, to look for additional details that confirm or falsify the conceptual context, then to construct a more general or more restricted context and so forth.

Examples in theories of physics are discussed in depth in part II but briefer illustrations are appropriate here. Examples of explication in ordinary life: focussing on a face in a crowd, identifying a country with its head of state, selecting a word to describe a feeling.

In physics, Newton's optical experiments were a paradigm of explication: defined pencils of light were targeted onto prisms in an otherwise darkened chamber and the experiments were crafted to select certain phenomena and to yield sharply defined results.

Explication through experiment is directed towards ideals of isolation, simplicity and constraint. Natural phenomena, such as planetary orbits, are amenable to scientific examination to the extent they approach the same ideals.

An important kind of explication is ***stabilization of the image***. A stabilized image has features (details) that are invariant as contexts vary. The eye picks out and follows a bird flying over a lawn. Progress in physics has been achieved through the identification of phenomena and concepts that can be stabilized, including conserved quantities such as momentum or mass-energy; prepared states such as atomic beams and their theoretical equivalent, solutions of Schr`dinger's equation that constitute a complete set of stationary states; and constants of nature like ***c*** or ***h***. Another form of stabilization involves extremal methods (e.g., the principle of least time). It is the premise of calculus that motion can be instantaneously stabilized on a continuous basis. The belief that all action can be expressed in such ***states*** expresses the desire to generalize stabilization concepts so as to comprehend physical reality.

Implication includes the construction of proposed physical laws and other conceptual contexts. Such laws can be tested against novel phenomena. Kepler's laws applied to planets actually observed and others later discovered. The periodic table suggested the existence of previously unknown elements. Belief in these contexts is strengthened by becoming incorporated in contexts of larger scale, e.g. Newton's dynamical laws and the system of protons, neutrons and electrons.

In the child, space and time are constructed in stages based on developmental adaptation of primitive concepts to more and more phenomena. (Piaget & Inhelder 1948; Piaget & Inhelder 1969) Physics has developed by progressive enlargement of implicative coordinate systems including those impossible to justify realistically, such as the complex probability amplitudes of quantum mechanics.

The employment by physicists of physically unreal concepts supports an instrumentalist interpretation. On the other hand, it is indisputable that physicists employ these concepts productively to apprehend, however tentatively, real phenomena and their actual successes support realism. In the indirect approach, the conflict is resolved by suggesting that reality is not entirely structurable but approximated by structural representations, even if reference to reality is problematical; and that domains of application are selective rather than comprehensive.

Mathematics is a structure of completely explicated forms. There remains the question of whether a physical entity or system can be completely explicated in general or even for a particular purpose. It appears possible, for example, to prepare an isolated neutral helium atom so that its features are completely explicable for all practical purposes. The same cannot be said of an isolated neutral U_{235} atom, where the moment of radioactive decay cannot be predicted. Quantum mechanics suggests that reality is not entirely explicable.

Quantum mechanical indeterminacy aside, a belief in comprehensive mathematical formulations presumes that reality is completely explicable, composed of particles and their interactions, and that any physical system can be reduced in principle to an explicitly stated description. Do reductionist approaches yield comprehensive results that are exactly true? A belief in these approaches is supported by scientific experiments on physical systems that have themselves been explicated in practice by refined and powerful techniques. Such techniques beg the question as to a positive answer, but still leave it open to a negative answer. If the most powerful and refined techniques of explication fail to wholly achieve their goals, then the reductionist program is subject to serious doubt.

This essay examines evidence of the inexplicable based on analysis of incongruence appearing in precisely those aspects of reality where explication has been most successful, namely theories of physics based on highly explicated systems. Such evidence strikes at the heart of a belief in the comprehensive power of mathematics and in a direct view of homogeneity between experience and reality. A contrary hypothesis, that reality is not entirely structurable, seems to the author to provide the best account for such evidence.

§4 Structuring Incongruence

Part II closely examines congruence and incongruence in physical theories. Simple conceptual examples of incongruence help motivate the presentation.

(1) The "Necker cube" uses 12 lines to indicate a skeletal cube in two dimensions through a drawing on a page. When the two-dimensional drawing is

conceptually implicated in three-dimensional space, the imagined structure can be seen oriented in one direction or another, but never both together. In one orientation, four particular lines indicate the face nearest the viewer; in the other orientation, the same four lines indicate the face farthest from the viewer.

(2) "Verbal description of a visual image" requires coordination of incongruent structures. E.g., write a description of the room where you are reading this essay. The visual image is grounded in three-dimensional space; all elements are simultaneously present; each element is specific and concrete; and the image typically displays an inexhaustible richness of detail. The verbal description is a one-dimensional series of words; most words are general abstractions (e.g., common nouns); and the description employs a finite, relatively small number of words. I have set forth the characteristics of the two structures in parallel to highlight incongruities.

While writing a description, there is a mental back-and-forth and the writer will explicate features of the visual image while implicating the corresponding words in the verbal structure, revising the description in stages as larger areas of the visual image are incorporated. Different individuals viewing a single scene will produce different verbal descriptions according to purpose, past experience or temperament. No verbal description is complete or comprehensive. Details can proliferate without limit.

(3) The concept of "map" is often used to describe the relationship between a model and the aspect of reality the model describes. Maps of the earth necessarily incorporate errors and distortions. “Not only is it easy to lie with maps, it’s essential. ... Maps have three basic attributes: scale, projection, and symbolization. Each element is a

source of distortion. ... There's no escape from the cartographic paradox; to present a useful and truthful picture, an accurate map must tell white lies." (Monmonier 1991)

Different projection methods generate different distortions. Hence, two maps of a single area of the Earth, a particular nation, say, based on different projection methods will almost but not quite coincide. They may agree, however, as to some particular parameter, both being "equal-area projections." Distortion is minimized through choice of a particular method and focus of projection based on the features of the area to be mapped.

If we had a collection of plane maps but were ignorant of the spherical nature of the original and of the projection methods used to generate the plane maps, the incongruities between maps would be mysterious. This situation can be compared to incongruities among multiple theories of physics where, according to the indirect approach, an aspect of reality not entirely structurable can be approximated in different ways.

(4) Although all mathematical functions are based on a common form of definition, classes of functions are sometimes incongruent. For example, one class of functions is sinusoidals of period 2π/k, where k is an integer. Another class is "step functions" made up of segments each constant over an interval and collectively defined over a segment of the real line. An introduction to calculus typically uses step functions to approximate the area under a continuous curve.

Consider those step-functions where the segments are collectively defined over the real line from -π to π. Such step functions can be used to approximate the value of any sinusoidal of period 2π/k over the defined interval from -π to π. The approximation can be as close as desired.

Conversely, Fourier analysis can use the class of sinusoidals of period 2π/k to approximate functions constructed over the real line from -π to π, including step-functions that are collectively defined everywhere on that interval . A particular Fourier approximation of order K is a weighted sum of the members of the class of sinusoidals where k≤K, and Fourier approximations can be calculated to approach any step-function in the limit as K grows indefinitely large.

Hence sinusoidals can approximate step-functions and vice-versa. In general, a closer approximation requires the employment of more functions in the approximating set.

The two sets of functions are incongruent. For example, the derivative of a step-function is everywhere 0 except at a finite set of points and step-functions are not everywhere continuous. In contrast, every Fourier approximation is everywhere continuous and has 0 derivative only at a finite number of points. The incongruities persist no matter how close the approximation.

This analysis foreshadows analysis of the theory of the critical point discussed in § 6(c) below.

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These examples of incongruence illustrate general features that reappear in physical theories: progressive development; proliferation of detail, more detail yielding greater exactitude; adaptation for particulars; layering of incongruent structures atop a common substrate; and incorrigible incongruities.

Part II Analysis of Theories of Physics

The principles of part I are applied to two areas of physics. The argument of the essay is that congruence supports epistemic homogeneity while incongruence supports epistemic heterogeneity. Slightly incongruent theories evidence more accurate approximations than highly incongruent theories.

Relativistic gravity (including space, time and mass) is the best candidate for homogeneity: there is apparently an exact fit between theory and aspects of reality. The thermodynamics and statistical mechanics of (1) rarefied "ideal gases" and (2) critical point phenomena exemplifies heterogeneity: each theory is exact only in the sense of a limit and theories are incongruous to one another. Another area of physics, negative electronic charge or "theory of the electron," also supports heterogeneity, but that topic is beyond the scope of this essay (see Mac Gregor 1992 and Hestenes & Weingartshofer 1991). Additional support is provided by problems that appear intractable, such as turbulence.[2]

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[2] "There is a good approximate theory of turbulence, the Kolmogorov theory, and it is scale invariant. But this theory cannot be quite correct, because it assumes that turbulence is homogeneous. In reality, a turbulent field always shows clumps of intense turbulence in a relatively quiescent background (this is true at all scales!)" (Ruelle 1991, 74) Even with supercomputers, calculations of turbulence are a "Computational Bête Noir" and "would take several thousand years to compute the flow [of air over an airplane] for one second of flight time!" (Moin & Kim 1997)

§5 Gravity, Space, Time, Mass

The three stages of development of gravity nest neatly within one other: Galileo's $s=\underbrace{\quad}gt^2$, Newton's inverse square law and Einstein's gravitational field equations. In retrospect, Galileo's formula applies near the surface of a radially symmetric mass (like the Earth) and Newton's law is the weak field limit of the gravitational field equations. Each earlier formulation is implicated simply and directly within the more general case.

Each formulation proves itself with the most highly explicated system consisting of two bodies reducible to points. The only consequential quantity is mass (first defined by Newton). Relativity theory generally posits conditions such that space-time-mass-gravity can be separated from other physical phenomena. E.g., that gravity causes effects on a test particle or electromagnetic wave, but not vice-versa. (See Møller 1966, §§ 110, 115-117)

Relativity theory appears homogeneous to such isolated aspects of reality and to provide exact descriptions of space, time and gravity over every scale of observation, from sub-atomic to inter-galactic distances, from the smallest subdivision of a second to billions of years and for masses of every size. There is only one formulation. There is no need to modify, particularize or reformulate the equations to adjust for diverse contexts. The equations are rigorous. They facilitate results expressed in stabilized forms such as geodesics. Although series expansions may be useful to solve particular problems, e.g., precession of the perihelion of Mercury, (Møller 1966, §130), this is a matter of convenience only.

In contrast, and with features described in parallel with the preceding paragraph, formulations of quantum mechanics are inherently indeterminate and are actually applied only in the sub-microscopic domain. There are two major formulations (Schrodinger and Heisenberg) and a demonstration of their equivalence requires a system of transformations based on arguments employing infinitesimal limits. (Dirac 1958, §§ 27-28) Every actual problem in, e.g. the Schrodinger formulation, involves the insertion of a particular "potential" and the construction of a particular Hamiltonian. The paradigmatic cases involve time-independent (stabilized) potentials. "The formulation of approximation methods for time-dependent phenomena is rather difficult, and ... depends on a straightforward expansion in terms in powers of the perturbing interaction. Not all interactions are weak, however, and in certain circumstances even weak interactions can lead to large transition probabilities." (Gottfried 1966, 423) For scattering problems, there is "no practical way of computing the[] functions" and approximations are required, with various choices available. (*Id*., 387)

The "renormalization" methods of quantum field theory are of particular importance because they are involved in the most successful theory of critical point phenomena in thermal physics discussed below. These methods were developed to deal with serious problems in quantum mechanics and have never been entirely satisfactory. "This is a blemish which cannot be avoided in our present state of ignorance..." (Dirac 1958, 308) "There is actually one swindle here... One usually waves one's hands here..." (Gottfried 1966, 405) "...pathological features. These always appear in the form of infinities or ambiguities at certain stages of the calculations. ...inherent in the structure of the theory." (Jauch & Rohrlich 1976, 170) "'Renormalization' ...is what I would call a dippy process! ...hocus-pocus... I suspect that renormalization is not mathematically legitimate." (Feynman 1985, 128)

Relativity theory supports the indirect approach in some aspects. The development of human spatial cognition follows a universal path, with stages of development involving divergent concepts, that culminates in the construction of a single unifying context within which all spatial phenomena can be organized, the "intuition" of Euclidean space. (Piaget & Inhelder 1948, 449-456) Relativity theory demonstrates that this intuition, deemed indisputable for more than two millennia, is erroneous. After such a demonstration, no constructed context can be wholly trusted.

Some relativity theorists desire to bring all events into a single context called "spacetime." But the question of whether "spacetime has an existence independent of its contents" is still controversial. (Norton) Belief in such an independent existence leads to serious problems, but denial allows for "merely different mathematical descriptions of the same physical reality ... [that] agree on all observables." (*Id.*) Such a denial, and the indirect approach, are consistent with the statements of Einstein (1916a, 117; 1916b, 161) that his theory "takes away from space and time the last remnant of physical objectivity" and that "space, time and event ... are free creations of the human intelligence, tools of thought, which are to serve the purpose of bringing experiences into relation with each other, so that in this way they can be better surveyed."

§6 Thermal Physics

Discussing "paradigms of emergent behaviors," (Lebowitz 1999) espouses a direct approach: "Statistical mechanics provides a framework for describing how well-defined higher-level patterns or behavior may result from the nondirected activity of a multitude of interacting lower-level entities. The subject was developed for, and has had its greatest success so far in, relating mesoscopic and macroscopic thermal phenomena to the microscopic world of atoms and molecules."

"Emergent behaviors" are a mainstay in the belief in comprehensive formulations and a homogeneous view. It is believed that chemistry "emerges" from physics; physiology "emerges" from chemistry; psychology "emerges" from physiology — all "emerging" into a comprehensive structure. The indirect approach challenges these beliefs.

Some comments on thermal physics supporting a heterogeneous view are conveniently gleaned from Stuart, Gal-Or & Brainard (1970, 1, 509, 510) and Gal-Or (1974, 435, 436, 439):

"It is amazing to note the conflicting opinions expressed by eminent scientists." (I. Prigogine)

"We all seem to have a different, a private congenial way of justifying the First Law, etc., and argue about the rationale in each separate formalism." (J. Kestin)

"Thermodynamics is something which develops, which expands, which grows, and it has the capability of growing, and this kind of growing is just like the house that Jack built, by patching on and patching over and mending, and so this is the reason, I believe — the historical reason — why there are so many differences in deriving thermodynamic properties." (O. Redlich)

"The motivation for choosing a point of departure for a derivation is evidently subject to more ambiguity than the technicalities of the derivation. Motivation is tied up with psychological and philosophical factors, and these are nowadays not considered *bona fide* topics for public discussion." (L. Tisza)

"I hesitate to use the terms 'first law' and 'second law', because there are almost as many 'first laws' as there are thermodynamicists, and I have been told by these people for so many years that I disobey their laws that now I prefer to exult in my criminal status... The term 'entropy' seems superfluous, also, since it suggests nothing to ordinary persons and only intense headaches to those who have studied thermodynamics but have not given in and joined the professionals. (C. Truesdell)

"...(entropy) is a property, not of the physical system, but of the particular experiments you or I choose to perform on it." (E. T. Jaynes)

"...there cannot be a rigorous mathematical derivation of the macroscopic equations from the microscopic ones. Some additional information or assumption is indispensable. One cannot escape from this fact by any amount of mathematical funambulism." (N. G. van Kampen)

As differentiated in Muschik (1993), there are a multitude of formulations of entropy in thermodynamics, including those of Clausius, Kelvin, Duhem, Caratheodory-Born, Falk-Jung, Serrin-Silhavy, Landsberg and Truesdell.

With these issues in view, some selected topics in thermal physics lead toward conclusions that progressively take shape.

a. The First Law of Thermodynamics

The first law of thermodynamics is a statement of conservation of energy that was empirically and historically based. (Kuhn 1959) It is an exemplar of explication to be contrasted with statistical mechanics, discussed below, an exemplar of implication.

The course of explication, illustrating the general principles of Part I, is conveniently set out in Sprackling (1991). The study begins with "an appropriate region of the universe that can be effectively separated" that is called the "***system*** ... The remainder of the universe outside the system is known as the ***surroundings***..." (Sprackling 1991, 4)

When "the bulk properties of the system are well-defined and, where appropriate, uniform throughout the system, so that each coordinate [e.g. pressure] has a single value that does not change with time, the system is said to be in ***thermodynamic equilibrium*** and its state is called an ***equilibrium state***." (*Id*., 7)

It is necessary to define a "***pure work interaction***" where, "if what happens in [the system and its surroundings] could be repeated, ... the sole effect to each is the change in height of a weight above a reference level." (*Id*., 25)

"Joule's measurements showed that, for the system to go from an initial equilibrium state i, with coordinate values p_i [pressure], V_i [volume], θ_i [temperature], to a final equilibrium state f, with coordinate values p_f, V_f, θ_f, by the performance of work only, always requires the same amount of work." (*Id*. at 27)

Joule's experiments were the original evidence for conservation of energy. Continuing to the present day, any and all direct evidence for conservation of energy is based on explicated systems. Nowadays, the principle is presumed and systems are built around the presumption.

Accordingly, in classical thermodynamics, energy conservation is predicated on explicative operations including a closed system and equilibrium states. Certain conceptual elements can be modified and "opened up," but the explicatory features of system v. surroundings and equilibrium states remain.

Statistical mechanics, on the other hand, presumes that a system is a collection of points, each point perfectly explicated. That is, each particle is presumed to be an infinitesimal closed system, imbedded in its own system of coordinates.

Every treatment of statistical mechanics is based on a Hamiltonian function that is the equivalent of energy. (Balescu 1995, 3) Statistical mechanics is invariably based on ***conservative*** force potentials between particles; and the Hamiltonian function is then a state, not varying in time. (See, e.g., Tolman 1938, §12). Hence, in statistical mechanics, conservation of energy is definitional and tautological.

It is, therefore, disingenuous to declare that the first law of thermodynamics is ***derived*** from statistical mechanics. What statistical mechanics shows is that, in a completely explicated system where energy conservation is presumed, it is possible to interpret work processes so that conserved energy expressed in work is at the expense of internal energy. This is a considerable achievement for the systems under consideration, but it is very different from providing support for a statement that thermodynamics "emerges" from statistical mechanics.

b. The Second Law

"Entropy," the subject of the second law of thermodynamics, is a term shrouded in obscurity. Viewing from the indirect approach, it is appropriate to inquire whether this obscurity is a result of an attempt to impose structure completely on a nature that is not entirely susceptible to such an imposition.

In thermodynamics, the state function of entropy is constructed through concepts of quasi-static (infinitesimal) steps and reversible processes, constituting further explication.

In kinetic theory and statistical mechanics, explication is presumed, and entropy is constructed by processes of implicative reasoning, which we now explore. An ***implication***, as discussed above, is the construction of a conceptual context around explicated details.

Both historically and pedagogically, statistical mechanics begins with a study of kinetic theory of an ideal gas "of N molecules enclosed in a box of volume V. The temperature is sufficiently high and the density is sufficiently low for the molecules to be localized wave packets whose extensions are small compared to the average intermolecular distance." (Huang 1963, 55)[3]

Ideal gases were also the original subject matter of thermodynamics and are described by the "ideal gas law" PV=nRT. The behavior of every gas is described by this law at sufficiently high temperature and sufficiently low density. The retreat from the ideal gas limit is accomplished through a mathematical series expansion in powers of N/V, called the ***virial equation***, i.e. $P/RT = (n/V) + B(T)(n/V)^2 + C(T)(n/V)^3 + \cdots$ that may, or may not, correspond to any actual physical reality. The functions B(T), C(T) are empirical constructs.

In an ideal gas, collisions between gas molecules are possible, but only binary collisions are considered (Huang 1963, 65-67) and it is assumed that the time during which a collision takes place is, in the limit, infinitesimal in comparison to the time

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[3] In his classic treatise, Boltzmann (1898) subtitled his general "Part I," "Theory of gases with monatomic molecules, whose dimensions are negligible compared to the mean free path."

between collisions. There is also an "assumption of molecular chaos," that "the velocity of a molecule is uncorrelated with its position." (Balescu 1995, 391 *et. seq.*)

This last assumption, (sometimes called Boltzmann's ***Stosszahlansatz***) "is in ***direct contradiction with mechanics ... [because] the interactions create correlations***." (Balescu 1995, 393, emphasis in original). The advantage of the assumption is that it makes it possible to formulate descriptions of systems displaced from equilibrium and of their "relaxation" into equilibrium states.

Development of the theory proceeds by a series of implications. For an ideal gas, it is possible to define a "distribution function" *f*(**r**, **v**, t) such that:

> "*f*(**r**, **v**, t) $d^3r\, d^3v$ is the number of molecules which, at time *t*, have positions lying within a volume element d^3r about **r** and velocities lying within a velocity-space element d^3v about **v**. ***The volume elements d^3r and d^3v are not to be taken literally as mathematically infinitesimal quantities.*** They are finite volume elements which are large enough to contain a very great number of molecules and yet small enough so that compared to macroscopic dimensions they are essentially points. ... Under standard conditions there are about 3 x 10^{19} molecules/cc in a gas. If we choose $d^3r \sim 10^{-10}$ cc, ***which to us is small enough to be called a point***, there are still 3 x 10^9 molecules in d^3r." (Huang 1963, 56, emphasis added)

In other words, this distribution function is a retreat from the completely explicated system and, in fact, such a retreat is necessary to define entropy. This is one line of implication, used in kinetic theory.

The line of implication used in statistical mechanics is somewhat different, through the construction of probabilities. A probability distribution function, *f*(**p**), "is the probability that an atom of gas will turn out to have a momentum vector **p** whose head lies within the volume element dV_p in momentum space." (Morse at 163). In such a formulation, likewise, dV_p cannot be a true infinitesimal quantity. Rather, probability functions support the construction of an implicatory structure of ***ensembles***.

Beginning with an actual system assumed to be in a definite completely explicated state, "[w]e may imagine a great number of systems of the same nature, but differing in the configurations and velocities which they may have at a given instant ... so as to embrace every conceivable combination." (Gibbs 1902, vii)

In addition, it must further be assumed that each such system is equally probable, an unprovable assumption about the behavior of details in the constructed context. Moreover, that the equality of probabilities will be further implicated in the evolution of the actual system so that the equal probability in concept corresponds to an actual equal probability in time — the ***ergodic hypothesis*** first expressed by Boltzmann and adopted generally thereafter. (Tolman 1938, §§ 25, 85; generally Farquhar 1964) There is yet another layer of implication in the construction of a phase space of 6N coordinates for a system of N particles, governed by Liouiville's theorem that prescribes conservation of volume for the system in this space. (Tolman 1938, §§ 19, 81).

There are important differences between kinetic theory and statistical mechanics. Kinetic theory considers systems displaced from and relaxing towards equilibrium and governed by assumptions such as Boltzmann's ***Stosszahlansatz*** while statistical mechanics, sometimes called "thermostatics," is strictly limited to systems in equilibrium states. Kinetic theory is restricted to gases, generally ideal gases and retreat therefrom through a virial equation, while statistical mechanics embraces liquid and solid systems.

For example, statistical mechanics can deal in generality with systems having more than one phase (e.g., liquid water and water vapor), a general treatment not within the scope of kinetic theory.

Seen from the indirect approach, both kinetic theory and statistical mechanics begin with presumptively explicated systems, but expressing different kinds of explication, the explication of ideal gas particles (or those deviating only slightly from ideal gases) vs. equilibrium systems. There are different premises and different idealizations. Development of entropy in the different theories requires different forms of implication, an incongruity. In particular, in statistical mechanics, and unlike kinetic theory, the original formulation could not express the ***increase in entropy*** that is predicted by thermodynamics. This requires an additional implication, called ***coarse-graining***, which builds around the exact energy presumed to be characteristic of the actual system an energy interval. (Farquhar 1964, 85). Coarse-graining is, therefore, a purely conceptual construct that "would make irreversability the result of our approximations." (Prigogine 1999)

Statistical mechanics and kinetic theory, therefore, construct systems of implications on top of assumptions devised for an ideal gas that include assumptions of uncorrelated interactions between particles, equilibrium states, relaxation from perturbations, the ergodic hypothesis and coarse-graining in an energy interval.

Once again, it is disingenuous to declare that entropy in phenomenological thermodynamics "emerges" from statistical mechanics. Entropy in kinetic theory and statistical mechanics is more restricted than the thermodynamic concept and requires implicatory machinery based on numerous assumptions, some of which are not physically supported.

Moreover, many of the assumptions of both kinetic theory and statistical mechanics are incongruent with conditions that occur in a "critical state" system that is at the polar opposite of an ideal gas as shall now be demonstrated.

c. The Critical State

The first definite study of "the critical state," published by Andrews in 1869, showed that a variety of fluid substances exhibited similar behaviors when subjected to a temperature and a pressure sharply defined for each substance (the "critical point"): the separation of the substance into liquid and gas phases abruptly disappears and light passing through a substance normally transparent is strongly scattered, a dramatic phenomenon called ***critical opalescence***. [Anismov *et. al.* 1995, Stanley 1971] For example, carbon dioxide, the substance most easily studied, becomes critical at 304.13 °K (near room temperature) and 7.375 atmospheres pressures. Corresponding behaviors were later observed among broad classes of systems including apparently all fluid-gas systems, magnets, alloys, polymers, liquid crystals, gels and foams, giving rise to the term "universality." [Agayan 1998]

It is possible to account for critical state behavior by general notions that are independent of the details of the system under consideration. Each particle in a system interacts directly with its nearest neighbors and indirectly with other particles more distant. Thus, the motions of any two particles are correlated through multiple pathways. "[T]he correlation between two [particles] along each of the interaction paths that connect them ***decreases*** exponentially with the length of the path. On the other hand, the number of such interaction paths ***increases*** exponentially, with a characteristic length that is temperature independent." (Stanley 1999, emphasis in original) At the critical point, the two effects exactly balance, and the influence of each particle extends throughout the system strongly affecting every other particle. A slight disturbance at each point will affect each other point and reciprocally. "Thus, a system at the critical point is characterized by ***correlations of infinite range***." [Balescu 1995, 319 (emphasis in original)] Away from the critical point there is no balance.

As the critical point, "the substance is no longer in thermodynamic equilibrium" because certain thermodynamic quantities are exactly zero and "for stable states," these quantities must be less than zero. (Anismov *et. al*. 1995, 1) Because of correlations that extend without limit, "both the amplitude of deviations and the size of the density-correlated domains in space increase" and "fluctuations grow anomalously." (*Id*., 72) It is no longer possible to define entropy, and "the asymptotic behavior of physical quantities is nonanalytic." (*Id*., 7, 72) Basic response functions such as compressibility and specific heat capacity become indefinitely large, i.e. approach infinity. [Stanley 1971, 25 *et. seq.*, 44] "[T]hermal perturbations do not relax for many hours or even days." (Agayan 1998) Hence, it is impossible to invoke an ergodic hypothesis which requires that the "time required for the measurement of a thermodynamical observable ... must [] be large compared with relaxation times for macrocopic variables of the system." (Farquhar 1964, 23-24)

From the perspective of the indirect approach, critical state behavior is both highly explicated, occurring only at a highly specific temperature and a highly specific pressure, but also completely implicated in that an infinitesimal change (a "fluctuation") in one part of the system may dramatically affect every other part of the system. The "universal" structure of ideal gases can be compared to the "universal" structures of critical state phenomena, but they are different "universal structures." The "universal structure" of ideal gases means that every gas has this structure as it achieves a state of complete explication and the "universal structure" of critical point phenomena means that every system has this structure as it achieves a state of complete implication.

The "universal structure" of critical point phenomena is based on the behavior of thermodynamic quantities near the critical point where the "leading term" (the predominating effect) takes on the form $X_0(\varepsilon^{-\lambda})$. [Stanley (1971), 40 *et. seq.*) ε is defined as $(T/T_c — 1)$ where T is the temperature where a measurement is made and T_c is the critical temperature. λ is a real number (e.g. 1.241) for all substances within various "universality classes," (e.g. fluids). The "amplitudes" X_0 depend on the thermodynamic quantity under consideration and the substance. (Anisomov *et. al.* 1995, 83) Examination of this form shows that thermodynamic quantities so specified increase without limit as the temperature approaches T_c. "Universality" is a consequence of the fixed nature of the λ for all substances within the universality class. It should be emphasized that this universality is only for the "leading term" of a series. As T deviates further from T_c, other "non-universal" effects predominate.

From the foregoing, it is possible to abstract a table of incongruities between two kinds of systems:

Classical Thermal Physics	Critical State
1. Commences with either (a) ideal gas or (b) equilibrium conditions and then proceeds through small deviations or infinitesimal steps.	1. Commences with conditions at critical point and examines small deviations therefrom.
2. Ideal gas or equilibrium state is conceptually explicated.	2. Critical state behavior is both highly explicated and completely implicated.
3. Equilibrium conditions are achievable and the core subject of analysis.	3. System never achieves equilibrium.
4. Fluctuations at one point are uncorrelated with fluctuations at any other point.	4. Correlations extend throughout the system. Correlation length is "infinite." Fluctuations "grow anomalously."
5. Small deviations "relax" toward equilibrium.	5. System never relaxes.

These incongruities lead to the conclusion that ***thermal physics is exact only at***

limit points and the conceptual foundations of theories describing the limit points clash irreconcilably.

Some physicists are conscious of the problems presented, although still pursuing the methodology of the direct approach. For example Fisher (1998, 653), in the vanguard of successful researchers into critical point phenomena, wrote (emphasis in original):

> "...when we have ideas and pictures that are extremely useful, they acquire elements of reality in and of themselves ... statistical mechanics [is] a theory ***not*** reduced and, in a deep sense, ***not*** directly reducible to lower, more fundamental levels without the introduction of new postulates. ... [I]t is possible to view the renormalization group as merely an instrument or computational device."

Detailed analysis of theories of the critical state is beyond the scope of this essay. The essential problem is that systems in the critical state display discontinuous behavior (e.g. infinite specific heat capacity and compressibility) and:

> "A slightly non-ideal gas interaction between molecules can be taken into account by perturbation theory, but as long as only a finite number of terms are considered, the continuity of the thermodynamic properties will not be destroyed. ***Discontinuous behavior can be introduced only by taking the perturbation series to infinity***... In fact the problem to be tackled in dealing with phase transitions is the 'strong interaction' problem in which the interactions can no longer be treated as small perturbations, but play a dominant role in the calculations and in the resulting physical properties." (Domb 1996, 9-10, emphasis added)

The first theory, that of van der Waals, using an expansion similar to the virial form, was proposed to account for Andrews' 1869 discoveries only 4 years later. A molecular field theory was proposed for magnetic phase transitions and the Ornstein-Zernike theory for correlations. [Stanley 1971] "Landau assumed that the free energy ... can be expanded as a power series ... about the critical point [where] any stable point must correspond to a minimum in [free energy]." (Domb 1996, 122)

> "[T]he van der Waals theory of a fluid, the molecular field theory of a magnet, and the Landau theory ... ***all give the same values for each of the exponents.*** ... However, it is clear ... that ***these fail to predict the observed values of the exponents***. Also failing in this regard are the Ornstein-Zernike theory and the various exactly soluble models..."
>
> [Stanley 1971, 48, emphasis added]

Because of "universality," examination of a stripped down model with highly simplified assumptions led to substantial theoretical success. The model was the "Ising model" for magnetism, where, by definition, each element of magnetism is fixed in space and the only interaction is between an element and its nearest neighbors. Onsager (1944) solved the two-dimensional problem exactly, but this solution required an assumption in which a plane was wrapped around a cylinder. Onsager's results did not correspond to the measured critical exponents, although this discrepancy was later accounted for by Wilson. (Domb 1996, 147)

The inspiration for further progress was the "scaling hypothesis .. independently developed by several workers." (Stanley 1999) Because correlations of all lengths are found in a critical point system, the situation should not change (in important respects) if the scale is (conceptually) doubled. And doubled again. And again. "The non-analytic critical behavior arises ... as a result of the limiting procedure as the number of iterations tends to infinity." (Domb 1996, 31) This was Wilson's "renormalization" approach that provided satisfactory results for a number of universality classes. It depended on the highly explicated features of the critical state because it progressively "pared" away all other states. In his Nobel Prize lecture, Wilson (1983) described the course of events in his personal and professional life, often adventitious, that led to his breakthrough.

Notwithstanding Wilson's *tour de force*, it is important to emphasize that his "solution" was of the purely theoretical Ising model. Moreover, "The renormalization group [RG] does not produce exact solutions of the Onsager type, and its application involves quite drastic approximations. ... No reliable method has been found of assessing directly the error at any stage of approximation of an RG treatment." (Domb 1996, 261)

Anisomov *et. al.* (1995, 108 *et. seq.)* explore the possibility of "splicing together" a "so-called regular region ... described ... by a virial-type equation of state" with the "irregular region [that] surrounds the critical point." "Naturally, when this approach is used, the resulting values of the critical parameters turn out to be different from the most reliable experimental values, and the critical exponents differ significantly from their analytic values. ... On the whole, the accuracy [] is poor..."

Moreover, a solution of "critical exponents" does not predict anything about other quantities involved in phase transitions. “So, here is a problem for theoretical physicists: prove that as you raise or lower the temperature of water you have phase transitions to water vapor or ice. Now, that’s a tall order! We are far from having such a proof. In fact, there is not a single type of atom or molecule for which we can mathematically prove that it should crystallize at low temperature.” (Ruelle 1991, 123-124)

§7 Conclusion

"Universal" theories account both for the behavior of systems in equilibrium and at the ideal gas limit and also for those at the critical point. Critical state behaviors are incongruent with theories formulated for the ideal gas/equilibrium systems and the incongruities appear irreconcilable. Seen from the indirect approach, the "universals" of the behaviors and of the theories are products of selected limiting points and the incongruities challenge beliefs in a comprehensive theory and in "emergent behaviors."

On the other hand, relativity theory appears to describe exactly the phenomena within its domain within enormously wide ranges and in a "universal" and unlimited fashion.

These polarities exemplify a condition of human life, that we are powerfully able to understand and control some aspects of reality but have only limited success in others. In others, partial general structuration is possible but beset by error.

This condition can be accounted for by a heterogeneity hypothesis that supposes that structures of experience are fundamentally different from the inherent nature of reality but that some aspects of reality are more or less susceptible of a structured representation, either in general or in a limit. The heterogeneity hypothesis also accounts for the development of multiple theories bearing on a single aspect of reality and for the appearance of radical conceptual changes in the history of science.

According to the indirect approach, it is not possible to predict beforehand which aspects of reality will be susceptible of structured representations or what form those representations will take. The researcher must select a problem for investigation on the basis of hint or hunch, carry a toolbox of representational forms that have been successful in the past, exercise judgment in applying the forms in particular circumstances, take advantage of unforeseen opportunities and depend on his or her capacity for spontaneous insight and invention. Attempts, failures and renewed attempts are the essence of the endeavor.

We have thus come by an indirect approach to the conclusion, towards which the author has throughout been striving, of having presented physics as an exemplar of freedom.

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