Advanced Note: Convolution devices -- a separate class of timing devices

1. ***Timing devices*** denotes a class of proposed devices that might be manufactured and used by electrical engineers and similar technology professionals. Timing devices are intended to mimic essential activity of neurons in brains. Assemblies of timing devices are intended to mimic essential activities of brain parts, e.g., using sensory inputs to control muscular action. One project is directed towards construction of an engineered organism that has six legs, along with equivalents of pressure and stretch sensors and equivalents of muscular bands. The organism would use a variety of muscular coordination patterns ("shimmering gaits") to find a path through, over and around obstructions in a terrain.

In approach and style, the system of timing devices mimics that of standard circuit theory taught in electrical engineering courses.

Standard circuit theory starts with a small "kit of parts" made up of simple devices, namely, resistances, inductances and capacitances. The devices are defined mathematically and are ideals; and they are also embodied in physical devices, namely resistors, inductors and capacitors that can be purchased and interconnected. The definitions for such devices are stated in terms of voltage and current of electrical signals passing through them, with "the same" definitions working mathematically and on the workbench. An engineer learns methods of construction to assemble circuits from the simple devices. After the engineer learns simple methods of construction, additional devices can be added to the "kit of parts," e.g., switches, transformers and transistors.

Similarly, the timing devices system starts with the set of devices set forth in *an Ear for Pythagorean harmonics*. Signals are made up of pulses. At this time, timing devices are only conceptual, but my intention is that the concepts should be embodied in physical devices that are manufactured and made available to engineers. The idea is that useful assemblies of timing devices can be constructed that mimic the activities of brains. Timing device constructions become a means for researching brains.

2. ***Convolution devices*** are a new class of timing devices that are different from and in addition to those defined in *an Ear for Pythagorean harmonics*, currently my standard statement of timing device principles. Convolution devices embody a method borrowed from electrical engineering, called "the convolution integral." Discussion of the convolution integral is beyond the scope of this note and I will presume that you have knowledge about.

3. Formal definitions: A ***timing device*** is a constructional element that engages in activity involving ***pulses***. As a "constructional element," a particular timing device participates in ***assemblies of timing devices*** that are constructed by an ***engineer***. The "activity" of a timing device is that it ***receives input pulses*** and ***produces output pulses***. A "pulse" is a uniform packet of energy that is received within a time interval (denoted "α") that is much shorter than all other relevant time intervals. Similarly, pulse production by a timing device also takes place within α. In comparison to all other time intervals, $\alpha \to 0$. The relationship between input receipt and output production is defined for each particular kind of timing device.

4. Begin with a timing device with one "projection onto" that carries input pulses to the timing device and one "projection from" that carries output pulses away from the timing device. There is a "laboratory clock" that provides a uniform time reference. Each pulse occurs at a ***pulse moment*** that is defined as an instant in time according to the laboratory clock. Let pij=tj where pij is the j-th input pulse and tj is instant of the corresponding pulse moment. E.g., tj=3' 2.00" on the laboratory clock. Similarly pok=tk is the k-th output pulse that occurs at time tk.

Let $\pi ij = pij - pi(j-1)$, where $pi(j-1)$ is the time of pulse prior to the pulse at pij; and
let $\pi ok = pok - po(k-1)$.

Each definition turns isolated "pulse moments" (the "p's") into a sequence of "pulse intervals" (the "π's").

In ongoing operations of convolution timing devices being constructed here, pok is the time when the next output pulse will be produced and po(k–1) is the time when the last output pulse was produced. The definition of πok means that, at the instant an output pulse is produced, the device determines the time interval leading to the next output pulse on the basis of the most recent input pulse or most recent input pulses. The determination by the timing device employs a method that looks backwards at the past, applied at the present instant. The rule is easily applied when determination of the next output pulse interval requires looking at only a single past input pulse interval. If we want to look at several recent input pulse intervals we need a "convolution" as constructed below.

3. Suppose

$\pi ij = \pi i(j-1) = \pi i(j-2) = \pi i(j-3) = \ldots$ That is, the input consists of a steady stream of pulses, separated by a uniform pulse interval, πij.

Then, suppose the timing device operates so that the output is also a steady stream of pulses:

$\pi ok = \pi o(k-1) = \pi o(k-2) = \pi o(k-3) = \ldots$

This activity is easily realized if πok = πij. Or, thinking along this line in a different direction, the timing device is ***defined*** by stating the rule πok = πij. The timing device so defined is a ***one-to-one timing device***. With a steady stream of input pulses, the device produces a steady stream of output pulses with the same pulse interval.

Although the approach to the one-to-one timing device was by way of a steady stream of pulses, the rule πok = πij applies even when the pulses do not arrive in a steady stream. Suppose, for example, the input is a slow steady stream of pulses that suddenly changes to a fast steady stream of pulses. The output of the one-to-one timing device will eventually operate "in synchronization" with the changed input pulse stream but the transition will take a moment or two. Many fast pulses may be received while the last slow pulse is being produced.

In developing and presenting the theory of timing devices, I follow a method that focuses on a kind of activity I call ***dwelling***. Dwelling is cyclical activity that does not change but, rather, exactly repeats. Dwelling in time is like setting tiles in space. Dwelling in timing devices is like a constant signal on a line in standard circuit theory. It is a point of origin for other activity. ***Silence*** is a central point of origin of dwelling and is like a "0."

[This Note is being written along with "Shimmering Silences in Beautiful Music," a major construction project for timing devices where various kinds of dwelling generate a class of phenomena I call "silences." Dwelling activity includes the "beat" of music, the "tonic" of music and "muscle tonus" in the bodies of animals.]

4. Define a ***time averaging timing device*** as follows:

$\pi ok = [1/M] \{\pi ij + \pi i(j - 1) + \pi i(j - 2) + ... + \pi i(j - M + 1)\}$. "M" is called the ***span*** of the average and states how far backwards the timing device is looking. The one-to-one timing device is a time averaging timing device where M=1. If M=2, πok, the interval between two output pulses, will be the average of a ***pair*** of intervals between two input pulses. A steady but irregular two-beat, -tə-DAH-tə-DAH-tə-DAH-, delivered as input, will result in a steady and regular output, dah-dah-dah-dah-. Longer spans result in more smoothing but may also show long term trends implicit in erratic patterns. A similar method in stock market analysis is called a "30 day moving average."

It is possible to conceive of applications for time averaging timing devices to control the activities of an engineered organism. A rise in activity that persists through time averaging can trigger action while a short rise will not, allowing for selection on the basis of long-term changes over short-term changes. One such application is for a system to "follow the beat" in music, for which, historically, I developed this line of devices.

5. The time averaging timing device is the simplest case of a convolution device – just as a "30 day moving average" is the simplest type of convolution integral.

To define the more complex form of ***convolution timing device***, re-constuct the definition of the time averaging timing device so that the interval until the next output pulse is based on a weighted average of a certain backward-looking span of input pulses, as follows:

Definition 1: $\pi ok = [1/M] \{dj \cdot \pi ij + d(j - 1) \cdot \pi i(j - 1) + d(j - 2) \cdot \pi i(j - 2) + ... + d(j - M + 2) \cdot \pi i(j - M + 2) + d(j - M + 1) \cdot \pi i(j - M + 1)\}$

The new terms in Definition 1 of the convolution device,

$dj, d(j - 1), d(j - 2) ... d(j - M + 2), d(j - M + 1),$

are coefficients. Each coefficient is a non-negative real number. The " · " means multiplication and the coefficients all start out as "1," in which case the convolution timing device reduces to the time averaging timing device. However, the coefficients can vary, so long as there is maintenance of an invariance constraint:

$dj + d(j - 1) + d(j - 2) + \ldots + d(j - M + 2) + d(j - M + 1) = M.$

Define a ***convolution function*** by stating its terms $c0, c1, \ldots, cM$. The c's match up with the d's, as shown below, and also collectively satisfy the invariance constraint, expressed as:

$c0 + c1 + c2 + \ldots c(M-2) + c(M - 1) = M.$

A convolution device is defined by stating its convolution function. The convolution function can be stated graphically as a function of time with c0 at t=0 and c(M-1) at t=M. Given the convolution function of the device and the set of input pulse moments up to a given instant, {pij}, the definition suffices to calculate the time of production of the next pulse.

In operations that take place over an extended period of time, a convolution timing device approximately realizes the characteristics of linear time invariance (LTI) that are of importance in standard circuit theory. Please note that the primal timing device discussed in other timing device presentations is not equivalent to an LTI device. There is no conflict. Implicitly the convolution timing device operates in a range of pulse intervals that are chosen to avoid operations that would violate linear time invariance.

Suppose we have three timing devices with different convolution functions, one with a peak near c0, the second with a peak in the middle and the third with a peak near c(M – 1). If the input pulse stream is suddenly faster, the output from the first device will be first to reflect the change.

A more advanced system of timing devices has signals made up of "pulse bundles" rather than a pulse stream. Such signals enable further use of convolution methods. Possibilities for further developments include timing devices with adjustable convolution functions.

To connect Definition 1 with the definition of the convolution function of a device, we need to match up the d's with the c's. As a practical matter, this means "flipping the convolution function" in a fashion learned by engineering students when they do convolution integrals. Accordingly, matching the defining values of the convolution function with the appropriate coefficients in the definition of the convolution timing device calls for:

$c0 = d(j - M + 1), c1 = d(j - M + 2), \ldots, c(M - 2) = d(j - 1) \text{ and } c(M - 1) = dj.$

Hence, when calculating the next output pulse with Definition 1, use:

$dj = c(M - 1), d(j - 1) = c(M - 2), \ldots, d(j - M + 2) = c1 \text{ and } d(j - M + 1) = c0.$