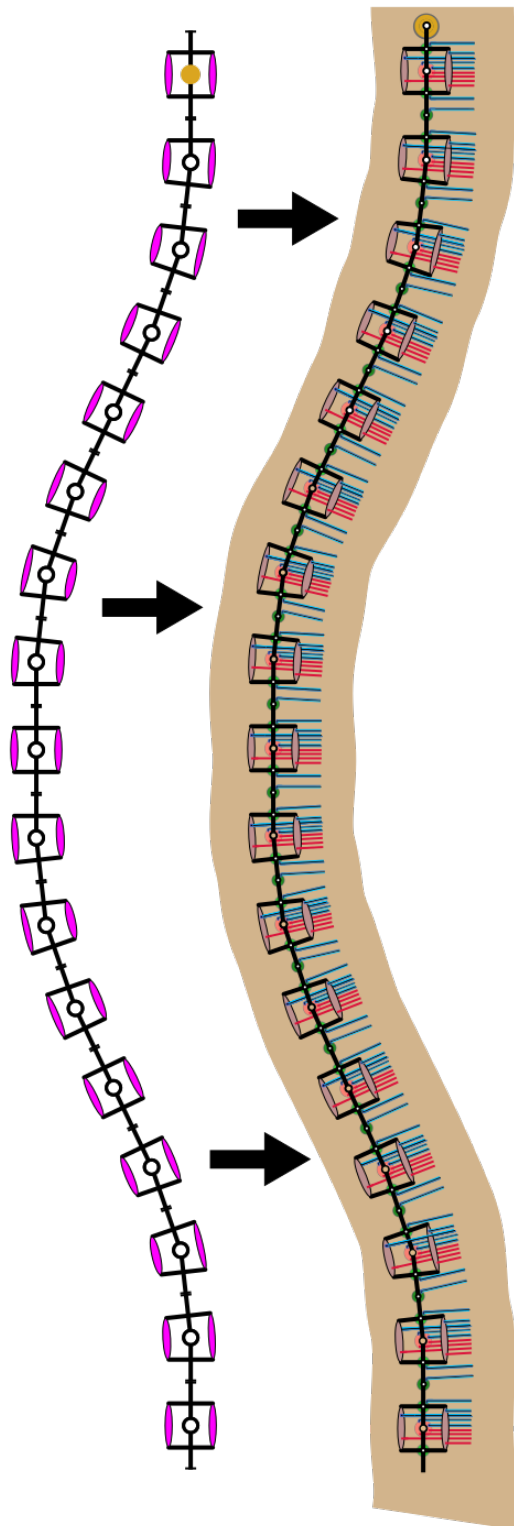


Metamorphosis of Wiggler I: device development for residential movements

metamorphosis of a spinal array



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The adjacent image shows changes in a spinal array based on the prior Wiggler I project. Unlike "remote movements" discussed there, developments here aim for production of body-based "residential movements."

Both arrays are in the shape of a waveform with a wavelength of 16 segments. Arrays have 17 segments for visual completion.

A waveform guides production of wavy movements but changing shapes of a moving array differ from a stationary waveform. E.g., forces in such a waveform are in equilibrium.

A new pair of annular movers encircles the central hub of each spinal segment, producing forces like those produced by exterior movers from the prior project. Forces from two pairs of movers can be added, subtracted and/or coordinated. To produce faster movements, movers are further multiplied in number and overdriven with expanded force differentials.

Red projections (lines) carry drive signals to movers. Two blue projections carry sensory signals from movers, denoting the length of a mover and changes in a mover's length.

Elastic joints, shown as green discs, replace rigid joints. Energy storage and release in elastic joints produces smoother movements. Blue sensors in elastic joints detect stresses (measured as deviations from equilibrium positions) and changes in such stresses.

A complete set of projections connects to control systems that reside in the body of the organism and that depend on the body. A duplicate set of projections connects to remote control systems in the head.

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Introduction

a. Limitations of Wiggler I point to needed improvements.

This project advances towards goals that were set forth in the prior Wiggler I project – chiefly, to design engineered organisms that produce movements resembling movements of animals, with many different repertoires of possible movements and with streaming selections of movements that vary according to momentary purposes, sensations, opportunities and risks. Ultimate goals include a network of control modules, each containing multiple quadnet devices that pass in synchrony through critical moments of Shimmering Sensitivity — as a multitude of possible movements change into a few selected actual movements.

When compared to such goals, movements produced by Wiggler I have serious shortcomings. A basic set of stationary positions is produced by opposing twitches and such positions do not move. Detached shifting movements between stationary positions are slow and stiff. Movements of independent joints are uncoordinated. Although sensory detection devices are anticipated, there are none in Wiggler I. An organism with such limitations is ill-suited to produce, e.g., wavy movements.

This project aims to overcome such shortcomings while maintaining successful aspects of the prior project: the Virtual Energy (VE) paradigm; classes of VE devices organized in kits of parts; assemblies of parts that operate in math-like domains; device control through pulse signals; whole-body control using a network of quadnet devices; and developmental principles of opportunism and invention.

Another aspect of the prior project is a broad generality that ignores important differences. Its principles apply equally to spinal arrays that are 1 meter long and 1 millimeter long. In contrast, if two actual arrays of such lengths were to be immersed in water, movements of the small array would be extremely slow compared to those of the large array and the large array would have vastly larger repertoires of movements. A small array can only move around in the water while a large array can also move water around itself. Similarly, tiny fish larva are powerless to resist a water current, while some adult fish swim upstream.

It is said that a large aquatic organism operates in a domain of *high Reynolds number* while a small aquatic organism operates in a domain of *low Reynolds number*. Distinctions between different Reynolds number domains cannot be ignored when residential movements are produced. A high Reynolds number domain is the chief focus here because it includes both slow and fast movements. Anticipated designs based on this project will produce the most sudden and forceful movements of the organism, namely, aversion movements, as well as wavy movements discussed here and movements of touching and self-touching.

- b. New designs aim to produce faster, smoother movements, guided by signals from sensory devices that detect internal positions, stresses and changes.

This project develops new versions of elemental devices used in the prior project, including new versions of projections, movers, bursters and spinal segments for assembly in arrays. New devices are expected to produce larger repertoires of movements with faster speeds and more flexibility than those of Wiggler I. Variable strengths and timings of movements will respond to sensory signals from detectors resembling proprioceptors in animals.

1. speed

Multiplicity of movers. In the prior project, a mover had two force fibers, making up a duet; and a spinal segment incorporated two opposing duets. In new designs, each mover has four force fibers, called a quartet. A quartet can operate both with two-beat schemata used in the prior project and also with new four-beat schemata. When two-beat schemata drive a quartet, a doubled level of force is possible.

Moreover, new "annular movers" produce repertoires of forces much like those of the "exterior movers" used in the prior project. Coordinated operations of two sets of movers can add, subtract or interweave force patterns. In anticipated aversion movements, fibers twitch collectively; the maximum force exerted, even without overdrive, is many times the force that was possible in the prior project. More force, of course, results in faster movements.

Overdrive operations refer to force productions outside the restricted set of forces there were defined in the prior project (§ II.B.3). Recall that the first design for opposing forces had 15 levels of force per mover; two opposing movers produced 29 stationary positions. In a restricted set, opposing movers produce only 15 stationary positions, centered around midline. The most rigid set of Group II drive signals was used in the prior project, namely:

(15, 8), (15, 9), (15, 10), (15, 11), (15, 12), (15, 13), (15, 14)
(15, 15) (midline)
(14, 15), (13, 15), (12, 15), (11, 15), (10, 15), (9, 15), (8, 15)

As an example of overdrive operations, suppose that a spinal segment is in the stationary position $k = 12$, maintained by drive signals (11, 15). A change in drive signals to (15, 10) will change the position to $k = 3$. For faster action, start with changed drive signals of (15,1) — producing the maximum force but outside the restricted range of equilibria. Then, as the angle approaches the final desired position, change again to drive signals (15, 10). This action presumes that there is sufficient time for device operations to sense the position, reduce the signal and bring the segment to rest, preferably without wobbling.

A further possibility looks to repetition of movements and adjustments of timings so as to approach the desired movement pattern, similar to practice and training.

Another design would use both sets of mover (annular and exterior) to change the angle of the joint in a spinal segment. The annular movers act as the accelerator of movement and the exterior movers act first as an accelerator and then as the brake. The fastest movement with this arrangement starts with maximum overdrive signals of all movers which are changed to reduced forces from annular movers and braking from exterior movers. Timing issues again become chief matters for investigation.

In another area where timing issues loom large, in wavy movements, alternations of force across midline are foundational; movers in a spinal segment start pulling to the left while joint parts are moving to the right and there is never a stationary position, only momentary pauses at the crest or trough of the cycle. Using only a single level of force, the same on each side, the size of back-and-forth movements can be controlled by the rate of alternation. For example, in the fastest mode of operation, a higher rate of alternation will produce smaller wavy movements.

2. flexibility of movements

Flexibility in new designs is based on elastic joints, which store and release energy and produce more complex movements. Hooke's Law ($F = -k \times \Delta x$) defines elastic materials and can be assimilated to the formula for VE force production in the force fiber, namely, $\mathcal{F}_n = (n \times \mathcal{F}_1) - (j \times \Delta L)$ (§ II.B.1 of the prior project).

Because terms using k and j have similar forms, elastic forces and forces produced by movers can be maintained in balances that shift in incremental steps. (Please see designs for Wavemakers and A Tube for Transport in the paradigms project.)

Stiffness k in Hooke's Law denotes both resistance to movement away from equilibrium and also energy storage in the material. Energy stored = $\frac{1}{2} (k \times \Delta x)^2$. Similarly, the dissipation factor j in the VE form denotes resistance to movement away from balancing positions — but there is no energy storage in the material.

During movements, elastic joints have flexibility that is absent in rigid-body designs of the prior project. In a shifting movement, for example, elastic joints slow down rotations of elbow joints and link joints during the early part of the movement, while energy is being stored; and then they speed up such rotations during the later part of the movement, while energy is being released. An undulatory quality can arise when successions of energy storage and release travel along a spinal array in coordination with changing mover forces. In further contemplations, it might be that body parts can be adapted to take advantage of undulatory properties of water.

3. sensory detectors and signals

As shown below, sensory signals can control movers directly in holding operations ranging over variable joint positions and variable lengths of movers. Different sensors have different physical sources and sensitivities, e.g., to joint positions or to mover lengths. All sensory signals have common forms and functions.

Joint positions and mover lengths are detected by sensors that, in stationary positions, generate a steady stream of pulses or silence. Frequencies of signals correspond to positions and are organized by a spectrum that varies from low to high. Here, an increasing frequency of pulses indicates a joint that is moving farther from midline or a mover that is growing longer. For other sensations, a higher frequency is a signal of a more forceful contact with an exterior object or a higher musical pitch. In another design (Eye for Sharp Contrast), a lower frequency signals a brighter light.

Changes in the length of a mover are detected indirectly from sensory signals by use of difference devices, which are a kind of timing device. Changes imply rates of change when device operations are specified. During operations of a "balancing unit," a difference device subtracts the index for the current length from that of the length that was detected a certain fixed time prior — and vice versa with another subtraction — and any difference in either subtraction results in pulsations. More pulsations signal a bigger change and also a faster rate of change.

Similar methods detect forces or stresses in elastic materials, which correspond to lengths in compressed or expanded materials by way of Hooke's Law. Changes in elastic stresses are detected by difference devices in designs like those that detect changes in mover lengths.

- c. Anticipated developments include body-based *extremum* principles, e.g., least energy flow, least time and least stress.

This project is focused on elemental devices. Spinal arrays that produce wavy movements are discussed in § 3.d but actual designs are deferred. More work is needed before a spinal array can produce smooth residential movements.

Anticipated developments in subsequent projects will address such tasks by methods adapted from topics in mathematical physics called analytic mechanics or calculus of variations. These methods investigate *extrema* such as maximum and minimum solutions to equations. Applications in physics include principles of least time, least action and least constraint. Adapted versions for VE devices include principles of least energy flow, least time, least momentary stress in a cycle of movements and least effort (stress accumulated over a cycle of movements).

The chief distinction between my methods and methods of mathematical physics is replacement of mathematical principles by actual repeated bodily movements with adjustments (practice and training). "Actual experimentation" is substituted for "mathematical experimentation" discussed below. The organism repeats a movement over and over, with slight variations in force strengths, timings, etc. Variations that result in more successful performance, greater VE efficiency, more speed and/or less effort (cumulative stress) are preserved in subsequent repetitions. Different *extremum* principles may compete for influence, leading to momentary balances that depend on the situation and on external circumstances. In a footrace, principles of maximum energy flow and maximum effort are most important.

In *The Variational Principles of Mechanics* at xxii-xxiii, Cornelius Lanczos sets forth "The procedure of Euler and Lagrange." He explains "the principle for one single particle. It can be generalized, however, to any number of particles and any arbitrarily complicated mechanical system."

... let us think of a particle which is at point P_1 at time t_1 Let us assume that we know that the particle will be at a point P_2 after a given time has elapsed. Although we do not know the path taken by the particle, it is possible to establish that path completely by ***mathematical experimentation***... (emphasis added)

... Let us connect the two points P_1 and P_2 by *any* tentative path. In all probability, this path, which can be chosen as an arbitrary continuous curve, will *not* coincide with the actual path that nature has chosen for the motion. However, we can gradually *correct* our tentative solution and eventually arrive at a curve which can be designated as the *actual* path of motion.

A mathematical quantity, the "action," is an integral (a kind of sum) "extended over the entire motion from P_1 to P_2 ."

It has a definite value for our tentative path and likewise for any other tentative path, these paths being always drawn between the same two end-points...

The value of this "action" will vary from path to path. For some paths, it will come out larger, for others smaller. Mathematically we can imagine that *all* possible paths have been tried. There must exist one definite path (at least if P_1 and P_2 are not too far apart) for which the action assumes a minimum value. The principle of least action asserts that *this particular path is the one chosen by nature as the actual path of motion.*"

As a preliminary matter, let's inquire whether the mathematical procedure is applicable to movements of wrigglers. Suppose that an actual spinal array of VE devices is immersed in water and its movers act according to signals provided by a researcher. Question: Is it possible to use methods of analytic mechanics and computation to predict the actual movements of the array from knowledge of properties of water, of signals and of mechanics of devices that make up the array?

The original mathematical principle had important limitations to its applications. According to Lanczos (see 348, 34 and 83-86): "The form in which Euler and Lagrange employed the principle holds only for the conservative (scleronomic) case," also called "time independent." In contrast, in cases called "rheonomic, i.e. time dependent, such a conservation law cannot be found."

Movements of wrigglers are time-dependent because energy is being dissipated and water is being heated. Conservation of energy is coupled with time independence and excludes dissipation.

Another limitation refers to "holonomic" and "non-holonomic" constraints; the original principles address only holonomic constraints. In water, dissipation depends on speed and changes movements in "non-holonomic" ways.

In *Classical Mechanics*, Goldstein discusses a more advanced version of the variational methods, called Hamilton's principle, and shows that, in certain cases "it is, thus, possible to extend Hamilton's principle to include nonconservative systems and nonholonomic constraints." "Another advantage is that the Lagrangian formulation can be extended easily to describe systems that are not normally considered in dynamics — such as the elastic field, the electromagnetic field, properties of elementary particles, etc." (pp. 38-45.)

Goldstein applies such extended methods to an electrical circuit. He asserts that such methods can also be applied to "a picture of a complicated system of masses on springs moving in some viscous fluid and driven by external forces."

(Goldstein, 45-46.) This picture describes something resembling wrigglers.

However, important aspects of wrigglers appear to be outside Goldstein's picture. In the system in his picture, it is possible for external forces to vanish; and the system then relaxes into a unique configuration of equilibrium, at rest and with an absence of dissipation. In contrast, forces are always active in wrigglers; there is no position of actual equilibrium; billions of stationary balancing positions have constellations of dissipation that depend on multiple factors, e.g., activations.

Goldstein's extension for dissipation involves the Rayleigh dissipation function that fits the form of the Hamiltonian method. This method does apply approximately to certain ranges of movements of inanimate bodies immersed in still water. But another kind of dissipation occurs inside a mover according to the term with j and this dissipation does not appear to be easily adaptable to the Hamiltonian form.

One approach to the inquiry would look first at the restricted sets of positions and movements of the prior project, namely, restricted to still water, stationary positions and detached shifting movements that are destined to come to rest. These would seem to be the easiest kind of movement to fit into a Hamiltonian picture. Even for this limited problem, there are serious problems to overcome, e.g., in fluid mechanics. Computational difficulties appear formidable. Problems and difficulties are multiplied when dealing with moving water and movements that depend on sensations — such as wavy movements and aversion movements.

Putting aside mathematical methods and difficulties, I suggest that certain actual movements of animals and people are guided by similar *extremum* principles that I call principles of least time, maximum efficiency, least stress and least effort. The clearest examples involve repetitive movements with slight variations. A bird at a feeder adjusts its posture during a series of pecks. While performing culinary tasks such as slicing and dicing vegetables, my own body similarly adjusts its posture. A basketball player practices shooting baskets from the foul line. Daily practice of such movements leads to improved performance, stronger skills, quicker movements and less stress. Trails left by animals in wilderness typically trace the quickest and easiest routes. We all cut corners: why go from A to B to C if you can go directly from A to C? We follow the path of least resistance.

Single-celled and other rudimentary organisms perform complex movements that appear to be based on *extremum* principles. Subash K. Ray *et. al.*, investigated the slime mold *Physarum polycephalum* and noted that such movements are seen in various simpler organisms:

- "Escherichia coli bacteria have been shown to select the best of multiple resources of varying quality."
- certain microorganisms "actively move toward locations with microclimates more favorable to their development."
- others "make compensatory decisions when faced with multiple conflicting sources of environmental information."

The slime mold produces even more astonishing kinds of movements:

P. polycephalum is a unicellular, multi-nucleated protist that can cover an area of over 900 cm² and move up to a speed of 5 cm/h. Despite lacking neurons, *P. polycephalum* shows complex decision-making behaviors. For instance, it can solve labyrinth mazes; form adaptive networks balancing efficiency, cost and fault tolerance, similar to those found in man-made structures; solve complex optimization problems; anticipate periodic events... [citations omitted.]

In an apparent system of operations, the slime mold has cyclical behavior that resembles operations of a quadnet, with a body containing a multitude of energy control centers and producing multiple modes of whole-body movements.

P. polycephalum's membrane is composed of multiple rhythmically contractile regions that lead to the emergence of a complex pattern of contraction-relaxation cycles at the organism level. The contractions occur about once every 60–120 s and result from the activity of the actomyosin protein networks that comprise the cell cytoskeleton. The membrane contraction-relaxation cycles are coordinated at the organismal level such that they cause the protoplasm to flow rhythmically back and forth throughout the cell, a phenomenon called shuttle streaming. The individual contractile regions change their contraction intensity in response to both the quality of the local environment and the contraction intensities of the neighboring regions (i.e., the coupling between the neighboring contractile regions). Previous studies have found that the contraction intensities in the slime mold *P. polycephalum* can change both in frequency and amplitude. When a region of a *P. polycephalum* encounters an attractive (e.g., a food source) or a repulsive (e.g., bright areas) stimulus in the local environment, the contraction intensity of the region increases or decreases, respectively. The coupling between the neighboring regions triggers a change in the pattern of membrane contractions throughout the cell, followed by the movement of the cell toward attractive and away from repulsive stimuli . . . (citations omitted.)

Bodily activities of the slime mold have a math-like application reported in *Popular Mechanics*, Thompson, "A Single Cell Hints at a Solution to the Biggest Problem in Computer Science." The Problem is the Traveling Salesman Problem, also called an "NP-hard problem." In the Problem, a salesman must visit a large list of cities and wants to find the shortest route of travel that reaches all of them. The solution to the Problem states the order in which cities are visited. The Problem can be solved by calculating all the possibilities but the time required for calculations increases very quickly when more cities are added.

The math-like application for a slime mold was developed by researchers at Keio University in Tokyo. They placed a slime mold in a device that provided attractive stimuli (food) and repulsive stimuli (bright lights) in a complex and variable pattern that could be interpreted as an instance of the Problem. So interpreted, the organism "solved" the Problem by movements of its body.

. . . the amoeba just reacts passively to the conditions and figures out the best possible arrangement by itself. What this means is that for the amoeba, adding more cities doesn't increase the amount of time it takes to solve the problem.

So the amoeba can solve an NP-hard problem faster than any of our computer algorithms. How does this happen? The Keio scientists aren't sure, exactly.

"The mechanism by which the amoeba maintains the quality of the approximate solution, that is, the short route length, remains a mystery," says lead study author Masashi Aono in a press release.

It thus appears that certain movements of rudimentary animals are guided by principles that resemble *extremum* methods of physics and computation. A process of production or a "mechanism" that explains the principles "remains a mystery." Notwithstanding the mystery, it appears to me that capacities of such a simple animal to embody *extremum* principles resemble, in rudimentary forms, more complex capacities displayed by higher animals, such as birds, and human beings. In a similar way, I suggest, methods of physics start with a single particle and lead up to complicated bodies controlled by pervasive principles.

Neither molecular mechanisms nor principles of computation have provided satisfactory explanations for movements of animal bodies that manifest *extremum* principles. As an alternative, I suggest a construction approach that proceeds with VE devices and collective devices. Collective devices operate in a device body that manifests whole-body or *endogenous* principles. As a simple example investigated in the prior project, a single input pulse causes changes in the whole

body of a quadnet device – and in all the bursters in the quadnet device. It is anticipated that further endogenous principles include *extremum* principles.

Suppose that a repetitive wavy movement of a spinal array of VE devices is driven by a source in the head. The head passes the signal to the first spinal segment, which shortly thereafter passes to the second spinal segment a signal that is based on the original signal but that is modified as result of signals from sensors in the first segment. Then the second spinal segment similarly passes a signal to the third spinal segment. And so forth down the line.

Focus on a particular spinal segment. During a cycle of movements of that segment, certain signals that drive movers and certain signals from sensors are aggregated in energy stores in the body of a device in the segment. The body has endogenous or whole-body operations that convert energy in such stores into control signals that are passed to the head and used to guide generation of new drive signals. "Now" drive and sensory signals are compared with recent past signals; and differences are reflected in the next set of drive signals.

Methods applied to signals for a single spinal segment also apply to a dynamic movements that are faster versions of the shifting movement investigated in the prior project, where a whole spinal array shifts from an initial configuration to a final configuration. A collective "whole body" contains and controls all the segments and segmented controls. Between the initial and final stationary configurations are a multitude of intermediate stationary configurations of the whole body. Dynamic movements can travel different routes guided by intermediate configurations. An actual dynamic movement does not reach an intermediate balancing position because new drive signals arrive before the position is reached.

Repetitive movements can be modified by means of training and practice. In a collective device body for the whole array, signals are aggregated during each cycle and endogenous principles control subsequent movements. "Actual experimentation" can thus be performed through variations in movement as sensed by the body.

1. Virtual Energy sources and distribution

a. The Virtual Energy environment

Every device, mover, burster, module or endogenous body is presumed to incorporate sources of Virtual Energy (VE) that fuel its operations. Operations involve conversions of VE from one form to another form.

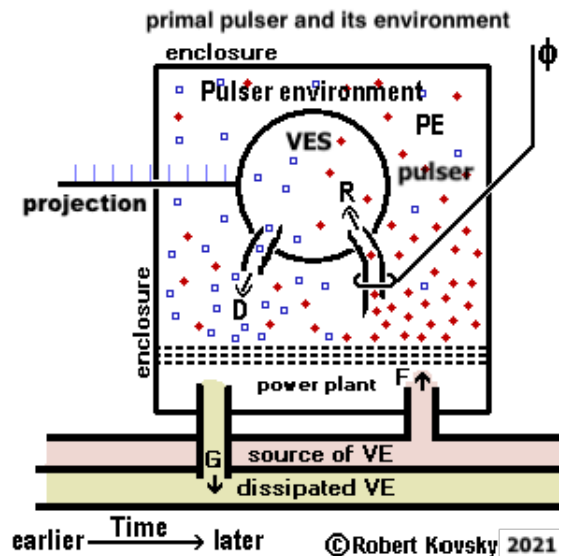
The figure below shows a design for the VE environment of a circular "primal pulser," adapted from one in the paradigms project. In a simple cycle of the device, VE received from a source is converted into VE stored in the Virtual Energy Store (VES) of the device; then VE in the VES is converted into pulses that are discharged onto a projection and that then travel to another device, where activity is triggered or affected. A new cycle is prepared with an inflow of fresh VE.

An enclosure surrounds the device; inside the enclosure, the pulser is immersed in its environment, denoted by PE. Within the enclosure but distinct from the pulser, a power plant is fueled by a source of active VE through a controlled channel, F — and it has an outlet G for dissipated VE.

Inside the enclosure, VE is carried by active VE particles, shown as red stars (*).

Active particles flow into the pulser through an inlet at a rate symbolized by R. In the pulser, VE is absorbed from active particles and stored in the VES. Dissipated particles, shown as blue boxes (□), are discharged through the port D.

The power plant takes in dissipated VE particles, re-charges them and returns them as active particles to the PE.



The inflow rate R is controlled by pulsestream ϕ , which is under the control of a researcher or generated as part of device operations. Something like a sphincter muscle encircles the inlet and the squeezing it exerts is specified by ϕ . In initial paradigms, ϕ is adjusted so that R is maintained at a steady or constant value and is always sufficient for desired device operations. The power plant is well supplied with VE; it can produce "more than enough" *'s.

As models of biological processes: the power plant resembles mitochondria; *'s resemble ATP; and □'s resemble ADP.

In anticipated large-scale designs, there is insufficient VE to maintain all devices at a high level of readiness and a new interpenetrating system of devices is used to control distribution of VE to various parts, modules, etc., by means of variations in the many ϕ signals. In a particular situation, some modules are fully fueled while others have reduced capacities. Which are active and which are inactive depends on the distribution of VE, which depends on the situation.

On such a large scale, I suggest that squeezing functions of the ϕ system resemble functioning of brain cells called *astrocytes* that are part of non-neuronal networks generally called *glia*. Glia cells are at least as numerous as neurons in a human brain and glial networks interpenetrate and interact with neuronal networks. Glia do not discharge pulses like neurons. They interact among themselves and support and control neuronal functions.

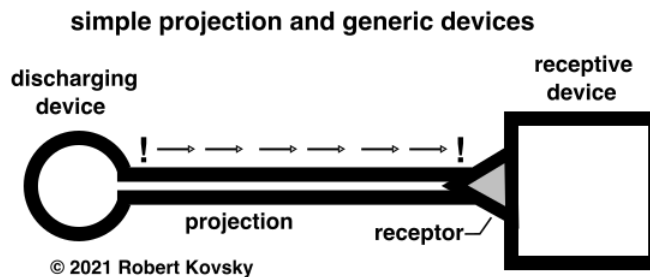
I suggest that networks of astrocytes and other glia cells perform functions in distributions of (blood sugar) energy to neuronal groups that are performing brain functions, e.g., controlling muscular movements. When a person is running a race, the largest share of internal nervous energy is directed to the lower body; when playing the piano, it is fingers, hands, arms and shoulders that are the most highly activated.

- b. Projections contain sources of VE.

Summary. Definitions for a simple projection and for more complex branching projections are stated in terms of Virtual Energy (VE) operations of devices. As in the initial Wiggler I project, movements of pulses on projections resemble both electrical signals in wires and action potentials in nerves.

(i). Simple projection.

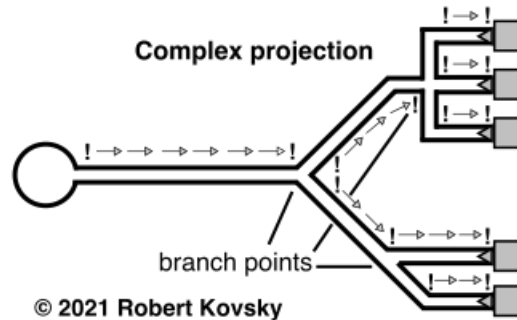
- (A) The design below uses very general parts: a simple **projection** carries pulses from a **discharging device** to a **receptor** that is part of a **receptive device**. A single kind of projection works with all devices (pulsers, bursters, movers, timing devices, quadnets) and with various receptors (see Fig. 31 in the prior project for applicable receptors.)



- (B) A pulse moves without change on the projection. The time required for such movement is much shorter than any other time period in device operations and is idealized as zero or **instantaneous**. Anticipated working models will use electrical signals.
- (C) Movement of a pulse on a projection is in one direction only. In current projects, movements on a projection are limited to pulses of a single size, exactly one bang (!) of VE. There is no loss of VE during pulse movement.
- (D) The projection can have any length and can be bent and stretched. Functioning of the projection is independent of its length or shape.
- (E) Similar to the PE of the primal pulser above, the projection has ample sources of VE in power plants in the discharging device and its own body that are more than sufficient to perform its functions.
- (F) A short period of time between pulses is required to restore the VE in a projection. This period of time, called an **iota**, is denoted by i . Iota is often used to label a short period of time required for certain device operations and the meaning can depend on the device. Here, i in projections is set at $i = .001$ sec. (1 millisecond); hence, the frequency of pulses on a projection cannot be more than 1000 pulses per second.

(ii). Complex projections and branch points.

- (A) Developing the prior definition, a **complex projection** carries pulses from a single discharging device to multiple receptors on multiple receiving devices. Pulses are multiplied at **branch points**, e.g., one pulse changes into two pulses. An example of a complex projection is shown below. As with the simple projection, travel times are idealized as instantaneous. Each pulse carries 1! of VE and there is no loss of VE during the transmission.



- (B) At a branch point, one pulse changes into two or more pulses. One pulse of size 1! goes into the branch point and two or more pulses, each of size 1!, come out of the branch point. The change occurs instantaneously.
- (C) It might at first appear that the device definition involves "new energy" at branch points, which would be contrary to principles of Virtual Energy (VE). However, like the simple projection, the complex projection has ample sources of VE in the discharging device and on its own that are more than sufficient to perform its functions, including pulse multiplication at a branch point.
- (D) Similar processes operate in axons of biological nerves, which propagate an action potential by means of internal energy conversions, e.g., in connection with saltatory conduction.
- (E) Idealized features of projections — instantaneous transmission of pulses and multiplication of pulses at branch points — recall idealized features of plane geometry — points without any sizeable dimension and triangles that can be mentally superimposed and tested for a perfect fit. In each case, I suggest, constructions in an independent imaginary domain can also lead to practical applications. Geometrical forms occur in space and do not change in time. VE forms occur in time; some temporal forms can be maintained or repeated, e.g., musical beats and melodies, rules stated in words — while others vary.

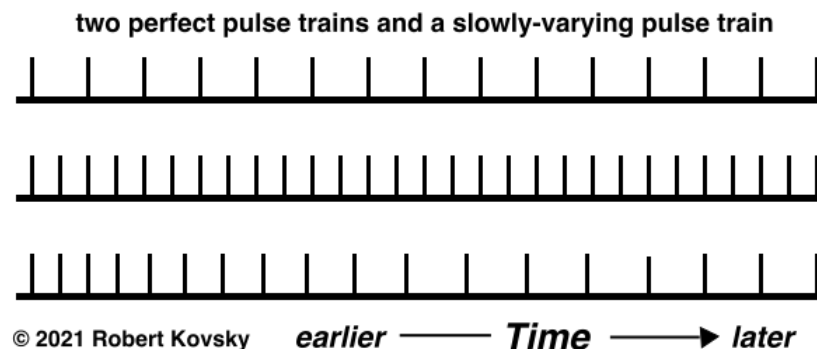
2. Sensory signals, detectors and modules

a. Sensory signals become inputs to bursters.

A sensory detector based on a pulser VE device was discussed in the prior project, § I.3.b . The output signal from the detector is a ***pulse stream***, a series of uniform pulses of size 1! with a well-defined period of time between each pulse and its successor. In the unified form constructed below, a range of pulse rates has a correspondence to both a range of sensation and also a range of motion. Linear correspondences are convenient here but other relations might also work.

An important special kind of pulse stream is a ***perfect pulse train*** where all periods between pulses are exactly the same. Imperfect or irregular pulse trains include slowly-varying pulse trains and gappy pulse trains discussed below. Pulse trains resemble tones in music and a perfect pulse train resembles a pure musical tone.

The figure below shows three pulse trains. Two perfect pulse trains have fixed periods; and one of the periods is twice as long as the other. The third pulse train has a period that slowly varies between the two fixed periods; slowly-varying means that a change between any two successive periods is no more than 10%.



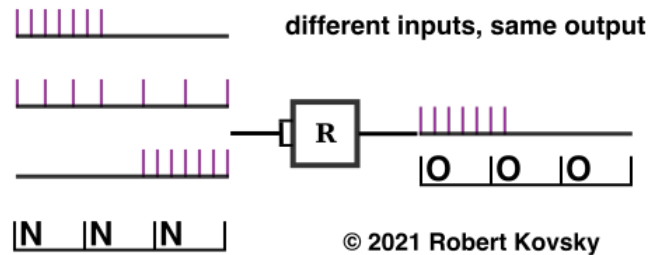
A pulse stream or pulse train is specified by a list of the periods of time between successive pulses. Sometimes, a pulse train can also be conveniently described in terms of a ***frequency*** of pulses, stated as a number of pulses per second or pulses per tick, similar to the number of beats per minute in music. Irregularities in a pulse train can impair or prevent application of the concept of frequency.

In residential designs investigated here, pulse trains are also used as inputs to bursters, replacing pulse bursts in many cases. This modification initiates a course of burster development, leading to the harmonic burster.

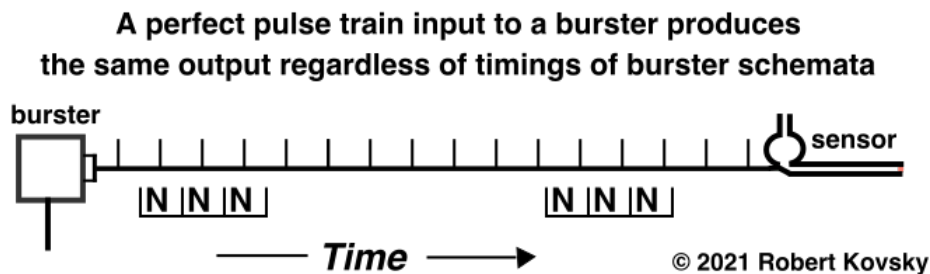
First, note that in the prior project, bursters were involved in multiple kinds of operations: ***ambient operations***, where the 8-tick schema of the burster starts on arrival of the first pulse in a burst; ***driven operations***, where a researcher starts the

schema; and *continuous operations* where the schema repeats without interruption. In the prior project, the different kinds of operations were not distinguished.

In the figure below, a continuously-driven repeating burster receives different input signals during the three N ticks of successive 8-tick schemata. The signals have the same number of pulses but with different times of arrival. The output is the same regardless of the timings of pulse arrivals. In all cases, 7 incoming pulses lead to the same V_b setting in the VES and the same output of seven pulses.



In the design below, a researcher starts the 8-tick schema of the burster at arbitrary times while a sensor device is discharging pulses in a perfect pulse train onto the receptor of the burster at the fixed rate of 1 pulse per tick. Regardless of when a burster schema starts, the receptor will receive 3 pulses in 3 N ticks and the burster will discharge 3 pulses during 3 O ticks. This rate is notated as 3!/3t.

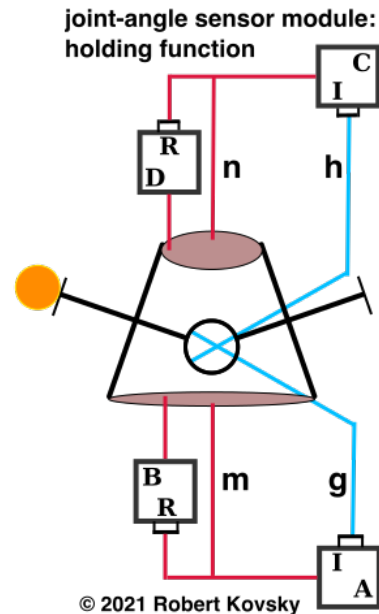


b. Sensory modules hold joints and movers at steady positions.

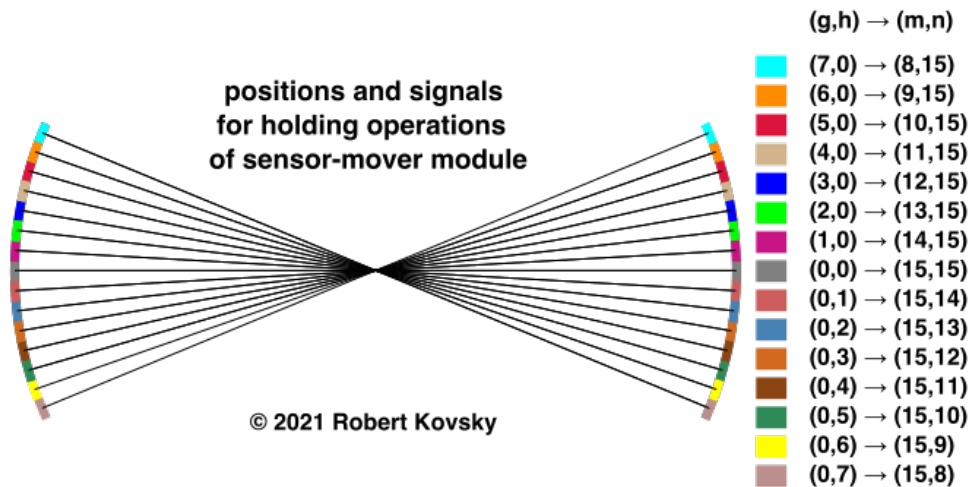
The joint-angle sensor module shown below "holds" the spinal segment in a particular position. If external forces move the segment to a new position and hold it there a few beats, the module will maintain that new position after the forces are removed. The movable position is called "steady" rather than "stationary."

The design resembles those in Figs. 9 and 11 in the prior project, with the same movers and spinal parts. B and D repeating bursters operate the same in all designs. A and C bursters, like those in Fig. 11, use inverting operations denoted by **I**. Bursters cycle continuously with 8-tick schemata. The orange disc identifies the head end and left/right (bursters A and B are on the left).

Two sensors operate in the joint; blue lines denote their output projections. A sensor is either silent or produces a pulse stream with a frequency that signals the angle of deflection from midline. One sensor (line g) detects deflections to the right; the other (line h) detects deflections to the left. If the joint is held at an angular position, the sensor generates a perfect pulse train; and sensory inputs cause bursters to hold that position.



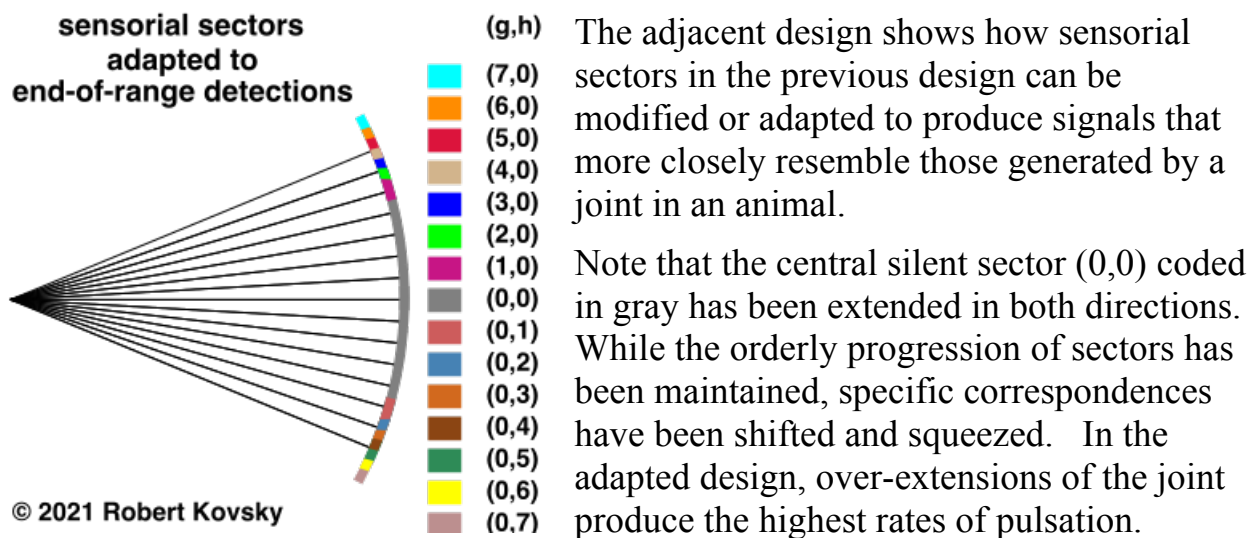
The position shown is $k = 11$ or $b = +3$. The g signal to burster A is $3!/3t$; the h projection is silent. The figure below shows the 15 stationary positions of the segment along with g and h signals produced by sensory detectors in the joint and resulting (m,n) drive signals to movers. The $b = +3$ example is colored primary blue. Each sensory signal has a frequency of $g!/3t$ or $h!/3t$. At midline, sensors are silent and drive signals are (15, 15). Rules here are $m = 15 - g$ and $n = 15 - h$.



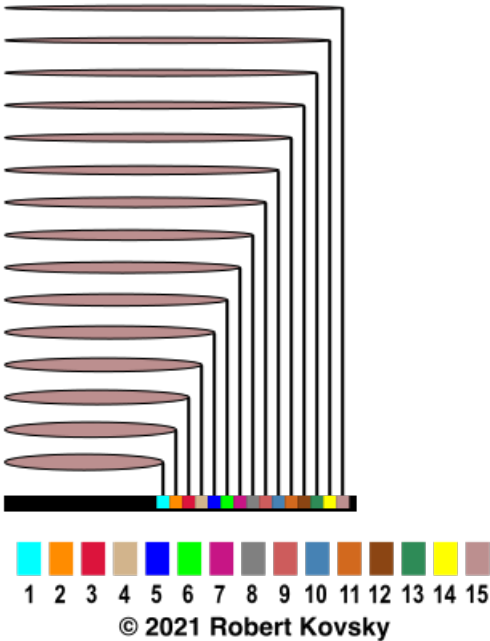
The range of movement is divided into 15 sectors coded by color. When the joint angle is within a particular sector other than midline, one sensor produces a signal with an integral number of pulses every 3 ticks. If external forces move the joint, the signal changes at the sector boundary, e.g., from $3!/3t$ to $4!/3t$. There is never a steady or continuing signal of $3.5!/3t$.

The primary function of this module is to hold a position. If a small external force is applied to the joint, resistance to movement will resemble that of an elastic material. If the force rises, however, the change in position will reach a sector boundary; and after a period of 2 or 3 beats, resistance will change to accommodation and the equilibrium position will shift one step in the direction of the external force so that the joint relaxes into the center of the next sector. A continuing external force can cause continued accommodation.

When compared to signals from sensors in animal joints, the foregoing model has a serious shortcoming. In an animal joint like a knee or an elbow, there are no signals near the center of the range-of-motion. Sensors in the joint produce pulse signals only when the joint is near an end of the range-of-motion; and pulses rates are very fast when the joint is forcibly over-extended outside the range-of-motion defined in the prior project.



detecting lengths of movers and linear sensorial sectors



In a further application of principles, the adjacent figure shows a repertoire of mover lengths used in stationary balancing positions. Sensorial sectors are shown for movers in linear opposition as in Fig 8 of the prior project. That is, lengths of movers in stationary positions fit a linear relationship. As with sensorial sectors for joints, sensorial sectors for movers can be modified to suit various needs, e.g., angular arrangements.

A sensor device in a mover produces a stream of pulses that varies according to the length of the mover. When the mover is at the shortest position, the sensor produces 1! every 3 ticks. When the mover is at the longest position, the sensor produces 15! every 3 ticks.

The adjacent mover-length sensor module performs a holding function like the joint-angle design; but using sensors that detect lengths of movers as shown above, with appropriate modifications of sensorial sectors.

Rules here are:

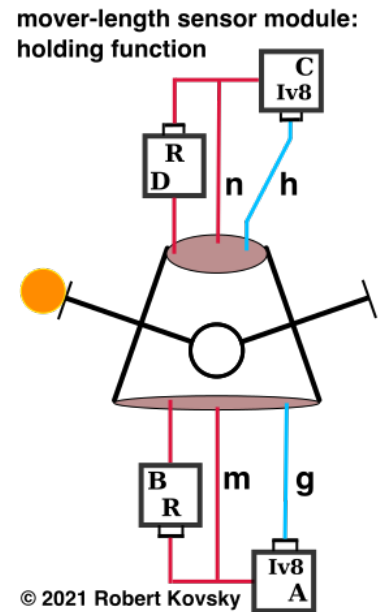
$$m = 15 - (g - 8) \quad \text{when } g \geq 8$$

$$m = 15 \quad \text{when } g \leq 8.$$

$$n = 15 - (h - 8) \quad \text{when } h \geq 8$$

$$n = 15 \quad \text{when } h \leq 8.$$

As before, g, h, m and n denote steady pulse streams with 1 to 15 pulses in 3 ticks. When the segment is held at b = +3 (3 steps right of midline), g = 11 and h < 8. Drive signals are m = 12 and n = 15.



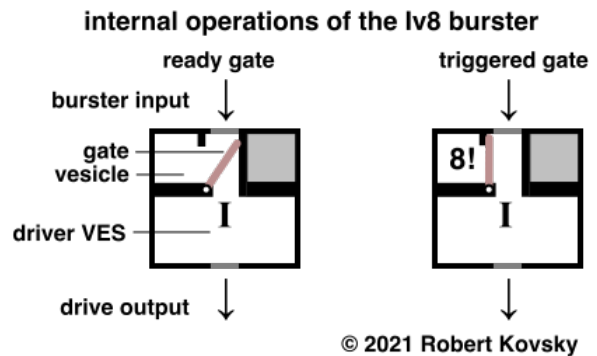
The Iv8 burster performs the same function as the combination of an m+ burster and an I burster in Fig. 11 of the prior project, namely, converting input signals organized by a left-right spectrum of positions into drive signals organized for an activation centered at midline. Two cycles of operations are required for a change of position in the prior design while only one cycle is required here.

c. Internal operations of the Iv8 burster

The Iv8 burster introduces certain internal features used in burster designs, namely, a *gate* and a *vesicle*. The gate closes and opens channels for the flow of VE. A vesicle resembles a VES in a device but a vesicle only stores VE without converting the stored VE into pulses. As the VES discharges pulses, any VE in the vesicle is dissipated; both vesicle and VES are cleared. VE is then restored in the VES for its operations but not in the vesicle.

Parts of the Iv8 burster shown below include the "driver VES" in the inversion burster I from the prior project, which produces the "drive output" pulse stream. New features are in the input parts.

The gate in the Iv8 burster has two positions, "ready" and "triggered." At the start of a cycle, the vesicle is clear or empty, the gate is in the "ready" position and VE pulses in the "burster input" are directed to the vesicle. If 8 pulses are received in the vesicle, the gate changes to the "triggered" position, and further pulses are directed to the VES.



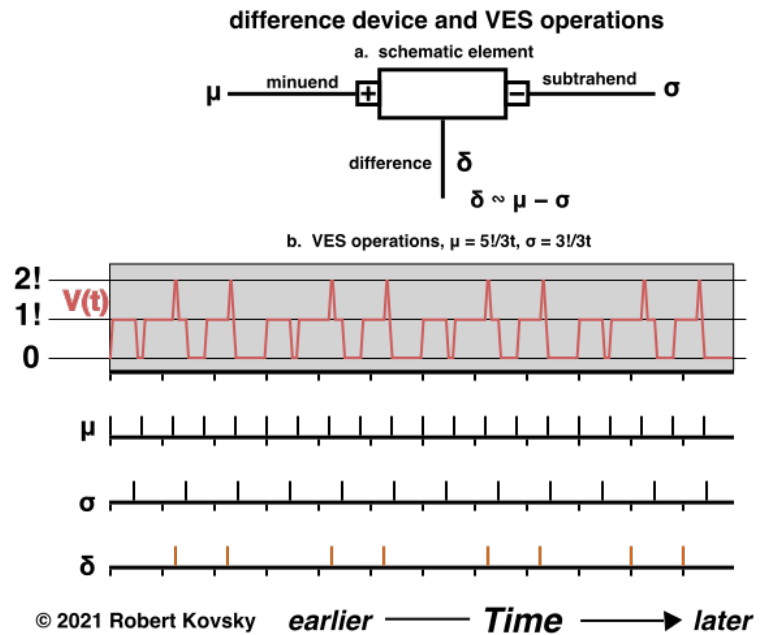
During the OOO ticks of the device schema, any VE in the vesicle is dissipated. The gate is returned to the ready position during the recovery tick R.

d. Difference devices detect changes in mover lengths.

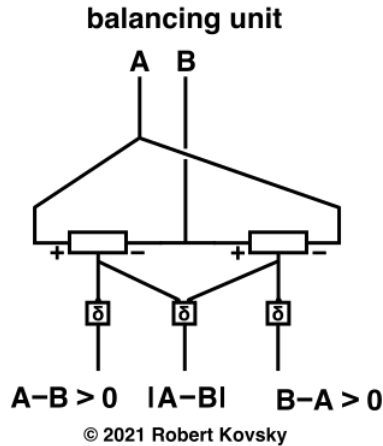
The image below shows the form of the difference device and a simple chart of VES operations. In the schematic figure a, a perfect pulse train, denoted by μ , with 1 to 15 pulses in each period of 3 ticks, travels over the "minuend" projection and arrives at the "+" receptor of the difference device. A similar perfect pulse train denoted by σ travels over the "subtrahend" projection, arriving at the "-" receptor.

The difference device produces an irregular or "gappy" output pulse stream δ over the "difference" projection with a regular frequency of $\delta \approx \mu - \sigma$, if $\mu \geq \sigma$. If $\sigma \leq \mu$, there is no output. In figure b, frequencies are: $\mu = 5!/3t$; $\sigma = 3!/3t$; and $\delta = 2!/3t$.

The chart of VES operations shows momentary changes in $V(t)$ — the level of VE in the VES. $V(t)$ ranges between 0 and 2!. If $V(t)$ reaches 2!, a pulse of 1! is discharged over δ and $V(t)$ drops from 2! to 1!. Each pulse arriving over μ raises $V(t)$ by 1!. Each pulse arriving over σ lowers $V(t)$ by 1!. If two opposing pulses were to arrive close together (not shown), the processes would cancel and $V(t)$ would end up where it started.



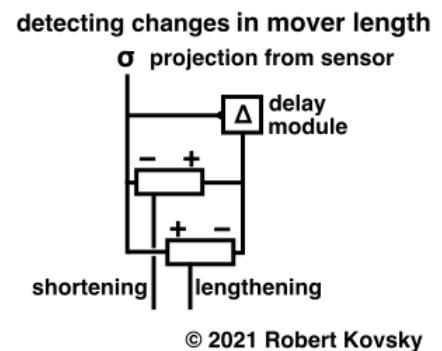
The difference pulse stream is constructed from the minuend pulse stream by removing or "canceling" certain pulses from the minuend stream and allowing the other pulses in the minuend stream to produce output pulses. Usually, a subtrahend pulse "cancels" the next minuend pulse. Occasionally, a subtrahend pulse arrives at close to the same time as a minuend pulse and it is that minuend pulse that is "canceled." Cancellations result in gaps. If there is a question of which minuend pulse is canceled, the difference between the two events is slight, shifting a gap one space forward or back.



The "balancing unit" shown in the adjacent figure is made of two difference devices. A perfect pulse train, A, arrives at the minuend receptor (+) of the first device and the subtrahend receptor (-) of the second device. A second perfect pulse train, B, arrives at the subtrahend receptor (-) of the first device and the minuend receptor (+) of the second device. If $A \neq B$, one difference device will produce an output.

Outputs of difference devices are connected to timing devices denoted " δ ." A timing device receives a pulse and produces a pulse after a short period of δ , e.g., $\delta = 0.1$ tick. The middle timing device produces pulses at a frequency equal to the absolute value of $A - B$.

In the adjacent figure, two difference devices and a delay module are used to detect changes in the length of a mover, based on changes in the pulse stream σ from the mover sensor described above. The "now" pulse frequency σ is compared to the pulse frequency that occurred a period of time in the past, with such period denoted by Δ . Suppose that $\Delta = 4, 8$ or 16 ticks.

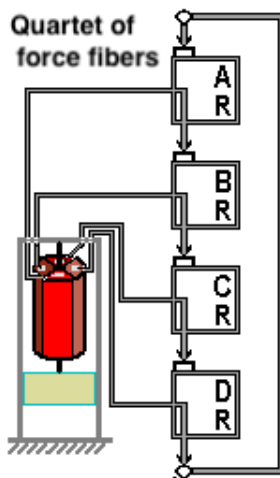


Changes in σ will result in output signals immediately if the mover is shortening, i.e., contracting; and after Δ ticks if the mover is lengthening, i.e. relaxing. Larger output signals denote faster changes.

The change detector is a kind of balancing unit with the now signal as A and the past signal as B. Thus the change detector compares two pulse streams from different times and the outputs denote any change. Similarly, balancing units can compare signals from the left side and the right side and outputs denote imbalances. Signals denoting changes or imbalances can be used to adjust movements and thus can tend to restore the prior condition or balance.

3. Unified wavy operations are maintained at low levels by quartet movers, harmonic bursters and delicate activations.
 - a. Quartet movers expand possibilities for movement.

Prior designs used duets of force fibers; two force fibers make a duet. Operations were based on two-beat schemata. Repertoires of movements can be enlarged by employing four force fibers in a module and by using four beats for burster schemata. The module shown below is called a quartet. Larger repertoires suggest that further multiplication of movers in a module may be fruitful.



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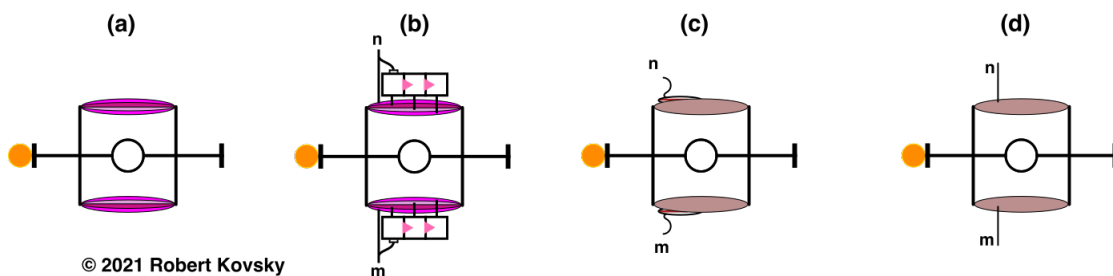
The adjacent figure, adapted from the bursters project, shows a quartet of force fibers; each of the four force fibers is driven by a corresponding burster.

In the prior project, two-beat schemata had the form abababab... (like a march or 2/4 time in music). Here, four-beat schemata have the form abcdabcdabcdabcd... (like common time or 4/4 time in music).

With a four-beat schema, the four force fibers hold a weight steady in a cylinder, similar to the design for a duet in Fig. 7 of the prior project. In this schema, each force fiber is relaxed and ready most of the time.

The figure below shows development of a spinal segment from the prior project with quartet movers instead of duets. Figure (a) shows movers with four separate force fibers. Figure (b) incorporates a burster module that drives each quartet. The first force fiber is triggered directly through the m or n projection during the first beat of force production. Successive fibers are triggered during three successor beats. A new drive signal is expected to arrive over m or n during the fourth beat of force production. Figures (c) and (d) show simplified versions for easier incorporation in designs.

development of a spinal segment with quartet movers and a burster module



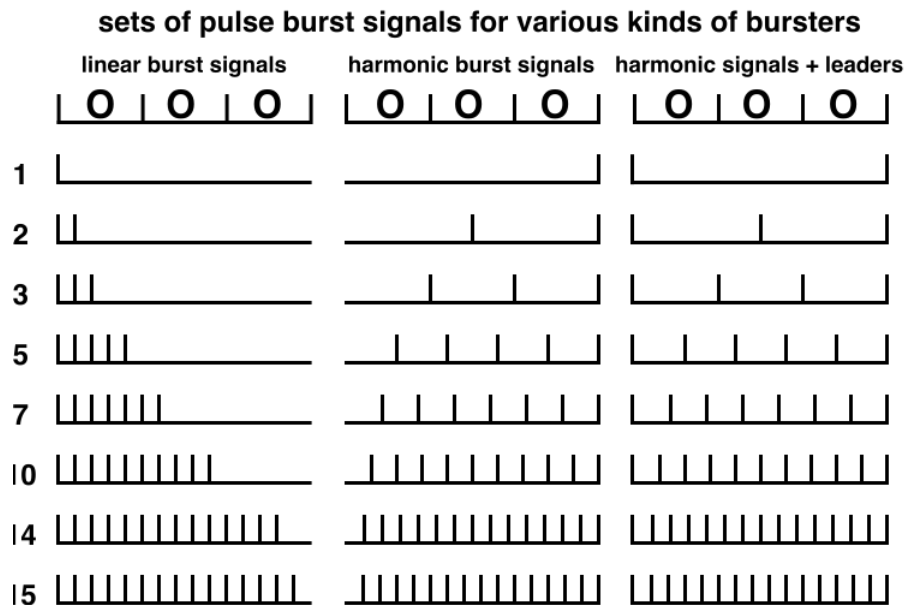
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- b. A simple set of signals unifies outputs from sensors and mover movements produced by harmonic bursters.

The figure below shows three sets of pulse burst signals that are used in various kinds of bursters. The signals are shown as outputs from bursters that occur during 000 ticks. "Linear bursts" were used in the prior project; and bursters from that project may henceforth be called "linear bursters." In new "harmonic bursters," both inputs and outputs use harmonic bursts. Sensors produce continuous pulse train signals that are equivalent to a repetitive series of harmonic burst signals.

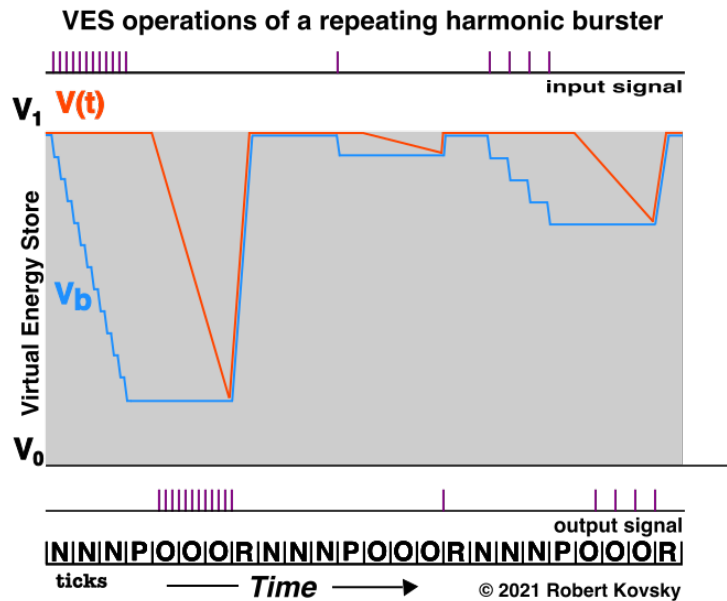
In terms of burster operations, the different sets serve the same primary functions. In current designs, changing the timing of pulses in an input burst does not cause any change in the output of a burster or a mover. A secondary distinction is that harmonic bursts that drive movers are congruent with corresponding sensory signals in certain device arrangements, such as the holding modules described above, thus providing a unified view of two streams of energy flow, perhaps useful, e.g., in aiming towards minimum-energy operations.

"Leaders" are initial pulses added to harmonic signals that provide convenient synchronization points and cues for the start of input operations. They do not cause a change in V_b in the VES. In harmonic signals with leaders, each burst lasts exactly three ticks, including both the initial instant and the final instant, with a tail that extends into the following P tick so as to complete the lowering of V_b .



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Modifications to the "linear burster" design in the prior project (II.C.1) develop into the harmonic burster. The figure below shows the new operations that occur during 8-tick schemata of a harmonic burster that is driven in continuous operations with an arbitrary input from the researcher. The bursts in the figure have 12, 1 and 4 pulses; there are no leaders.



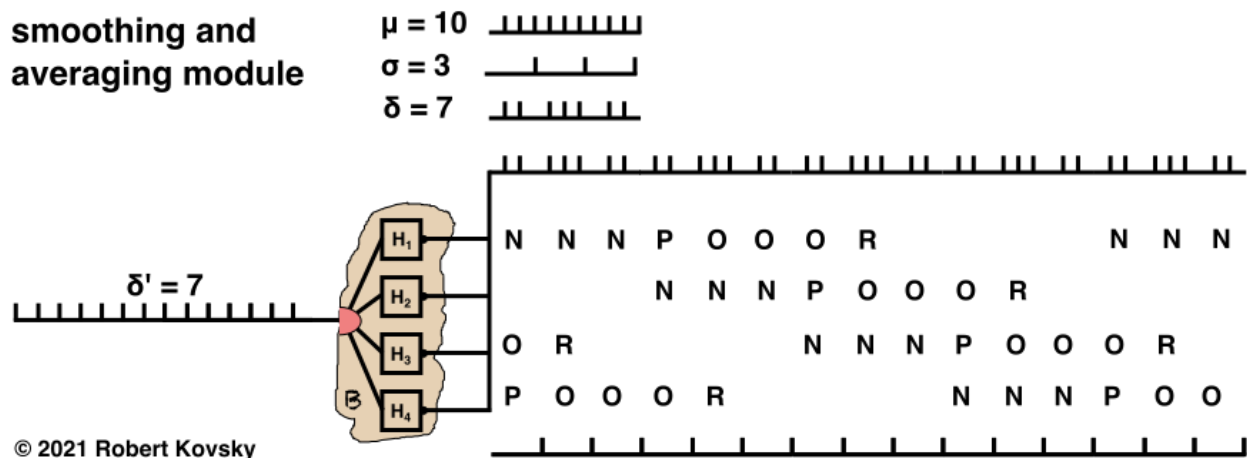
These operations resemble those of the burster shown in Fig. 10 of the prior project; but there are significant changes. As to shared features, both designs use the 8-tick schema NNNP|OOOR. Operations during NNN ticks have the same results in both bursters, regardless of different timings of input pulses and modifications in the lowering of V_b . No change is needed for operations during the recovery tick R.

The chief difference in operations occurs during the OOO ticks. In the linear version, $V(t)$ is driven down from V_1 to V_b at the rate of $15!/3t$ or $5!/t$. In the harmonic version, $V(t)$ is driven down at the rate of $(V_1 - V_b)/3t$. The device incorporates a repertoire of 15 such rates and the appropriate rate is set during the P tick according to V_b . The P tick also includes processing of the final pulse in the input burst; processing begins exactly at the start of the P tick and extends into the first tenth of the P tick. By way of lessening demands during the P tick, discharge of the VES starts exactly at the start of the first O tick in this design, while, in the prior design, discharge starts during the last two-tenths of the P tick. (Harmonic $V(t)$ operations resemble those of force fibers; see the bursters project.)

In the prior design, the first pulse in a burst starts the NNN bloc. In the simple harmonic design with driven operations, the last pulse in the burst occurs exactly at the end of the NNN bloc. When leaders are added to harmonic bursts, the NNN bloc is defined by pulses at both ends.

The figure below shows a design for a module that uses four harmonic bursters to smooth or average a "gappy" pulse stream.

The gappy difference pulse stream $\delta = 7$ is generated by a difference device with $\mu = 10$ as the minuend and $\sigma = 3$ as the subtrahend. To generate the δ stream, start with the μ stream and strike out or cancel any pulse that is preceded by a pulse in the σ stream or that overlaps a pulse in the σ stream. Three-tick units follow sequentially to make up a continuous pulse stream that serves as input to the module. Output from the module is in the form of the perfect pulse stream $\delta' = 7$.



Each harmonic burster H_i converts three ticks of the gappy pulse stream into a unit in a perfect pulse stream. As a harmonic burster starts its own P tick, it also starts the first N tick of the next harmonic burster. Ticking in the four bursters occurs synchronously through entrainment in the body \mathbf{B} .

c. The "most delicate activation" minimizes VE flows.

In § II.B.3 of the prior project, the initial plenary set of 29 balancing positions was restricted to a smaller number — 15 — that could be more easily controlled and that was sufficient for the purposes of that project. The restricted set was called "the most rigid Group II activation." There are two groups of activations (Group I and Group II) and each group contains 8 different activations. Of the 16 different activations, "the most rigid Group II activation" has the largest forces when the spinal array is in a midline configuration.

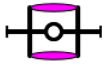






In this project, the polar opposite activation is employed, called "the most delicate activation." This Group I activation has the smallest forces when the spinal array is in a midline configuration. Balancing positions and drive signals are:

| | | | | | | | | | | | | | | | | |
|-------|-------|-------|-------|-------|-------|-------|---------|-------|-------|-------|-------|-------|-------|-------|--|--|
| | | | | | | | midline | | | | | | | | | |
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | | |
| (8,1) | (7,1) | (6,1) | (5,1) | (4,1) | (3,1) | (2,1) | (1,1) | (1,2) | (1,3) | (1,4) | (1,5) | (1,6) | (1,7) | (1,8) | | |

The adjacent figure shows stationary balancing positions of the spinal segments used in the prior project, along with drive signals, angles and bends of the most delicate activation. The figure is adapted from Fig. 19 of the prior project, which shows the most rigid activation.

The balancing positions in the two activations are identical and force differentials during a shifting movement between configurations should also be identical.

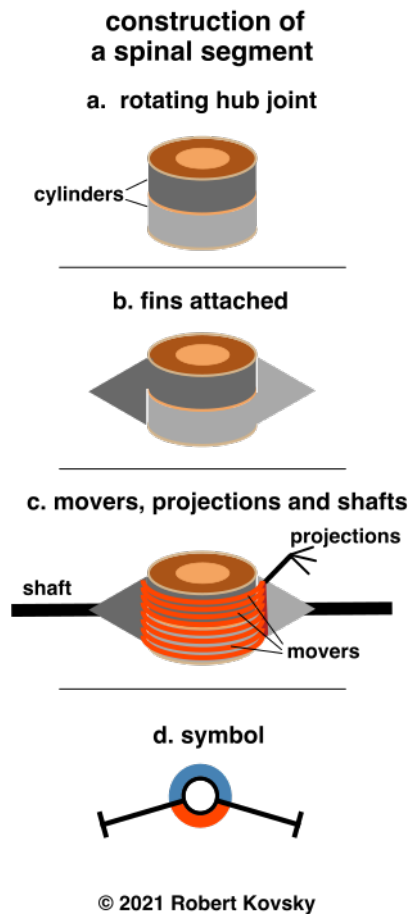
In this project, the level of activation is generally correlated with VE consumption. As to wavy movements, the most delicate activation consumes the least VE.

| Spinal segments and drive signals for the most delicate activation | | | |
|--|------------------|--------|-------|
| spinal segments | drive signals | angles | bends |
|  | 1 1 | 0.0° | 0 |
|  | 1 2 | 6.4° | 1 |
|  | 1 3 | 12.8° | 2 |
|  | 1 4 | 19.2° | 3 |
|  | 1 5 | 25.6° | 4 |
|  | 1 6 | 32.0° | 5 |
|  | 1 7 | 38.4° | 6 |
|  | 1 8 | 44.8° | 7 |

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- d. Variable activations suggest improvements in production of wavy movements.

4. Reconstruction of the spinal segment
 a. Central hub joint and annular movers











The new version of the spinal hub joint, like the original version, resembles the hub of a bicycle wheel. Two identical cylinders rotate smoothly around a common axis and without constraint. The central hole is reserved for brachial parts.

Symmetrical fins are attached to the cylinders. Fins are rigid planes, e.g., made of plastic. They provide anchors for movers and limit rotations.

Each force fiber runs from fin to fin with the belly lying on a cylinder. Two quartets of fibers produce symmetrized forces on one side of the fins and two opposing pairs of quartets operate on the other side of the fins. Attached shafts and projections round out the construction. A symbolic form for the joint is introduced (d).

Spinal segments, annulus movers and drive signals for the most delicate activation

| spinal segments | drive signals | angles | bends |
|---|---------------|--------|-------|
|  | 1 1 | 0.0° | 0 |
|  | 1 2 | 6.4° | 1 |
|  | 1 3 | 12.8° | 2 |
|  | 1 4 | 19.2° | 3 |
|  | 1 5 | 25.6° | 4 |
|  | 1 6 | 32.0° | 5 |
|  | 1 7 | 38.4° | 6 |
|  | 1 8 | 44.8° | 7 |

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The new hub joint produces bends from drive signals that duplicate those in the prior design. The image shows the most delicate activation.

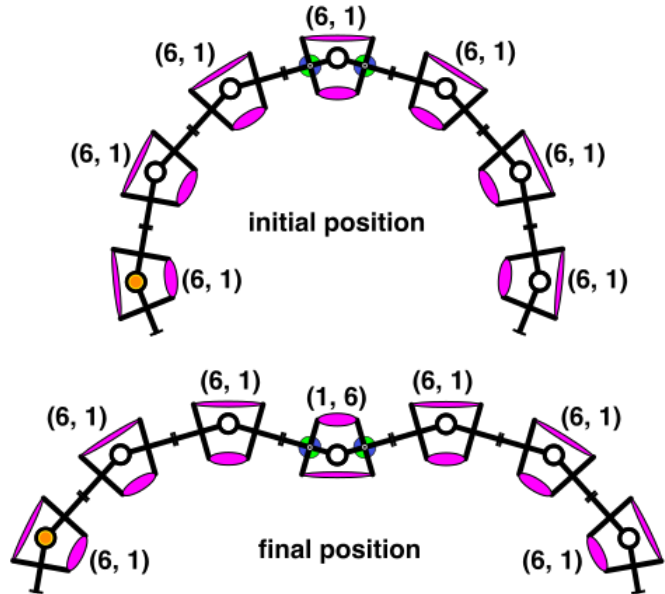
Annular movers conform to arcs of circles. Mover lengths in stationary positions have a linear relationship with the angles. Like the linear opposing duets in the prior project, the force relation has a constant dissipation factor j .

b. elastic elbow joint and exterior movers

The adjacent figure is based on the prior project and shows the initial and final positions of a shifting movement of a spinal array of seven segments. Only the central spinal segment changes its stationary position during the movement, from drive signals (6,1) to drive signals (1,6), a shift of some 64°

The central spinal segment has been augmented with elastic joints where the beams are attached to the shaft. The augmentation is without effect in the figure because elastic joints are in equilibrium when the array is in a stationary position and there is no deflection or restorative force.

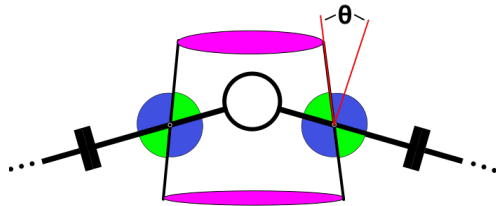
shifting movement of a spinal array with elastic joints augmenting the central segment



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stresses in elastic elbow joint

shafts remain momentarily at (6, 1)
while drive signals maintain (1, 6)

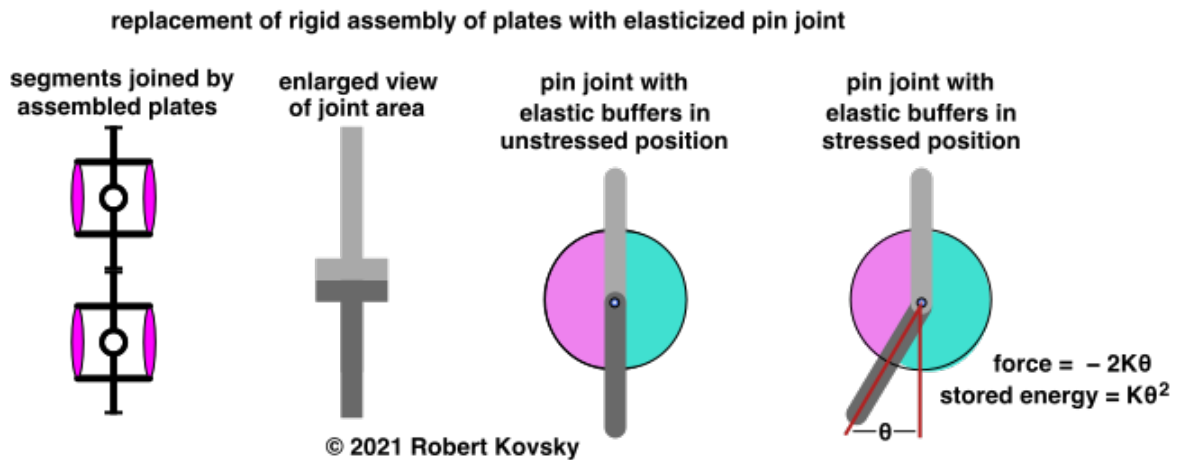


force = $-K\theta$
stored energy = $\frac{1}{2}K\theta^2$

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The adjacent figure shows the central segment a moment after drive signals have changed from (6, 1) to (1,6). Each shaft is rigidly connected to a massive series of segments that can only move slowly. The movers are directly connected to beams that can move quickly but with resistance from the elastic material. Resistance is low at first and a substantial deflection θ occurs. While there is a deflection, force is being transmitted from the beams to the shafts through the elastic material.

- c. elastic linking joints replace rigid assembled plates of spinal segments.



- D. detecting stresses and changing stresses in elastic joints

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