# Wavy movements in a hybrid domain of classical and Virtual Energy devices

by

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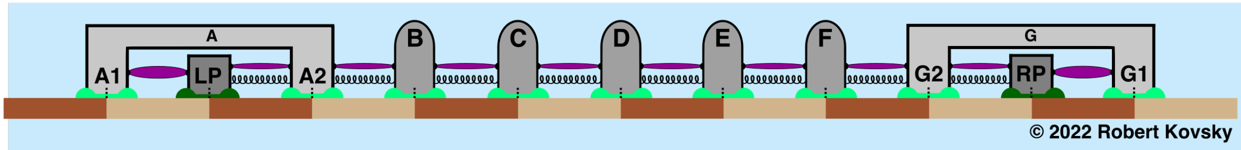
Wavy locomotion movements are produced by a linear array of masses and springs combined with Virtual Energy movers and glickers



Introduction

This project advances the program of development that started with the “wriggler project” and that continued with the “metamorphosis project” (*Wriggler I, an engineered organism* (2020); *Metamorphosis of Wriggler I: device development for residential movements* (2021)).

The program's long-range goals include designs for engineered organisms that can produce large repertoires of movements and that select movements on the basis of momentary influences arising from current goals and situation and from sensory modules that measure bodily conditions and detect external objects and events.

The original wriggler project constructed an imaginary Virtual Energy (VE) domain in which a linear array of modules (resembling segments of an animal's spine) produces classes of movements called "remotely-controlled movements" or "remote movements." In anticipated large-scale designs, such remote movements originate in a ***scriptorium*** in the head of an engineered organism and are represented in ***scripts*** prior to actual production. Scripted movements are unified with operations of VE control devices (“bursters”) and with symbolic forms resembling mathematical groups. The remotely-controlled movements of the wriggler project are detached from environmental conditions and detached from each other. Any individual movement in a series of movements can be delayed without changing the final result. Operations occur in ***detached time***.

The metamorphosis project investigated extensions of the VE domain so as to include various “residential movements,” especially wavy movements. Such movements would be produced by a ***sensorium*** in the body of the organism by means of operations that "reside" in the spinal array and that respond to ***sensory signals*** and other influences. Residential operations in the extended VE domain occur in a ***controlled time*** that mimics the ***actual time*** shared by material bodies that move and change. Designs for production of actual wavy movements were beyond the reach of the metamorphosis project and investigations therein focused on device developments that appeared to have potential use in reaching such goals.

Designs in this project advance towards residential goals. Arrays of device parts constructed below do produce wavy locomotion movements in a controlled-time VE domain. However, the approach is from a different direction. Here, designs start with a standard paradigm of wavy movements investigated in classical mechanics and mathematical physics — the Simple Harmonic Oscillator or SHO.

For purposes here, a “paradigm” is a simple, idealized mental construction or invention that operates in an imaginary domain and that can be used in practical applications. Geometry is an ancient exemplar. I have such a view of paradigms of physics such as the SHO, atoms, electromagnetic waves and conserved energy.

Device parts in electronics circuits provide more recent examples. Electronics parts purchased by working engineers and hobbyists perform as represented in specifications, within a range of approximation that may vary according to price (e.g., ±1%, ±5%, ±10%). Performance according to specifications does not depend on the researcher, time or surroundings. Thus, physics paradigms (mental constructions) are realized, approximately and in limited domains of application, by means of technologies, including devices, tools and manuals that define operations in words and symbols. Performance features of VE designs, e.g., in this project, are intended for similar realization in technologies.

Here, investigations start with movements of the SHO, which are shown in designs made of mechanical device parts: (1) small solid bodies (balls or blocks) with defined masses and (2) ***springs*** that manifest forces described by Hooke's Law and that are approximately realized by steel wires and rods bent into helix shapes. Friction and dissipation are initially excluded from the imaginary domain and then added back in the single form of viscosity of a surrounding fluid.

Then, in new developments, additional parts are added to classical paradigms, beginning with VE devices called "movers" that produce forces which drive movements. The result is a hybrid module that produces repertoires of movements described by classical physics. In locomotion movements constructed below, an array of device parts that resembles a segmented worm travels on a linear track with certain degrees of freedom, chiefly, direction, mode and size of a step.

Thus, movements in a classical physics domain are re-constructed as movements in an extended Virtual Energy domain.

Further anticipated developments in the VE domain aim at operations of bursters in modular and collective bodies; such operations control the movers that drive residential wavy movements. Designs in this project suggest approaches to such operations. A chief method is adaptation of principles from mathematical physics related to the SHO. Foremost are ***extremum principles*** and ***a principle of bodily experimentation***, discussed in the metamorphosis project. *Extremum* principles are borrowed from standard physics paradigms where maximum and minimum values of mathematical forms are called least time, least action and least constraint. In VE designs, anticipated *extremum* principles apply to repetitive cyclical operations of modules and bodies and include least time, least consumption of VE and least effort (cumulative stress).

The principle of bodily experimentation attributes operational values to conditions of the body containing VE modules rather than to mathematics. Variational methods of mathematics become variations in properties of the body. An experiment starts with a guess for initial operations that produce classes of results subject to a test or standard.  Perhaps the initial guess produces results that poorly approximate the standard and/or that work slowly and/or that waste energy.  Next, properties of bodies of modules are varied and various results are compared, choosing a "better result" as a step in further experiments.  The aim is progressive betterment.  New versions of modules and new kinds of devices are also welcomed in such experiments.  Similar developments have occurred in the technology of internal combustion engines, where the *extremum* principle or standard is "100% efficiency" in conversion of energy in chemical bonds in fuel into kinetic energy of a motor vehicle.

An ideal reader is familiar with SHO physics paradigms, as well as the wriggler and metamorphosis projects.  Constructions herein stand independently “on their own” but draw on these sources.  Concepts are chiefly expressed by means of figures and progressions of figures that make up the constructions.

Part A. Individual Oscillator

## § 1. Simple harmonic oscillator made of a mass and a spring

The construction begins with the paradigm of the ***simple harmonic oscillator*** (or ***SHO***), a fundamental model of physics with a heritage that stretches from Robert Hooke, through Newton, the Bernoullis, Lagrange, Fourier, Rayleigh and Schrödinger to the quantum mechanics used in semiconductors.

Construction of the SHO starts with an ***ideal spring*** attached to a fixed support, as shown in Fig. 1(a). The only property of the ideal spring is the ***stiffness*** **k** that controls movements. Springs vary in stiffness from "flexible" or "weak" (low k) to "rigid" or "strong" (high k) The ideal spring has no mass. There is no friction at this stage of development and gravity is ignored throughout.

In Fig. 1a., the spring is shown in a unique ***unstressed static position*** where no forces are present and nothing moves.

In Fig. 1b, the unstressed static position is denoted as "$y_0$." A metal ball is attached to the spring, with mass m and charge q. Although there is no gravity, the mass carries kinetic energy when the ball is moving.

**Fig. 1. Ideal spring in the unstressed static position**

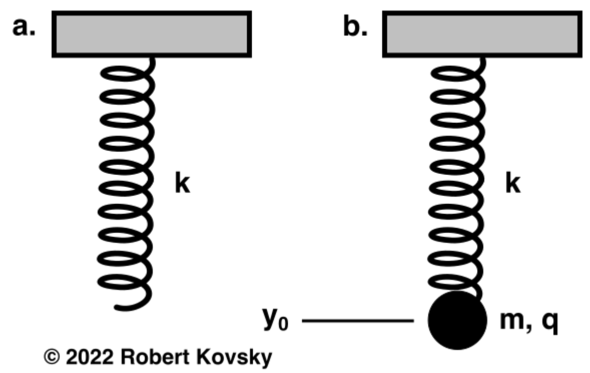
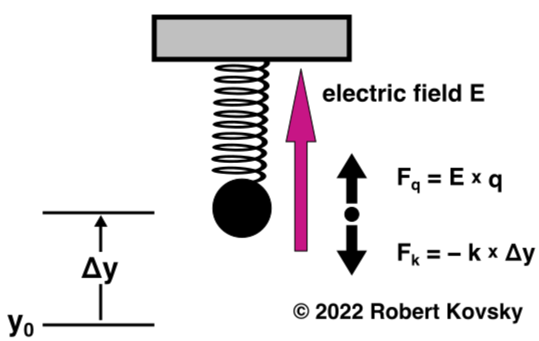



**Fig. 2. Ideal spring in a stressed static position**




As shown in Fig. 2, an upward-directed electric field **E** slowly increases from 0. **E** produces an upward-directed force $\mathbf{F_q}$ on the ball equal to **E** × q, causing the ball to rise. As a result, the spring produces a downward-directed force $\mathbf{F_k}$ with a size equal to k × Δy, where Δy is the change in position from the unstressed static position to the ***stressed static position***.

"$F_k = -\ k \times \Delta y$" or "$F_k = -\ k \times (y - y_0)$" is called ***Hooke's Law.*** The minus sign indicates that as Δy grows bigger upward, $F_k$ grows bigger downward.

The increase in E is so slow that the ball stops moving if the field stops increasing. (This is a ***quasi-static*** or "nil momentum" restriction, which vanishes when friction is introduced. Without the restriction, the ball jiggles perpetually.)

In other words, in a stressed static position, the net force on the ball $\mathbf{F_B}$ is equal to 0. In symbols, $0 = F_B = F_q + F_k = (E \times q) + (-k \times \Delta y)$ and $\Delta y = (q/k) \times E$.

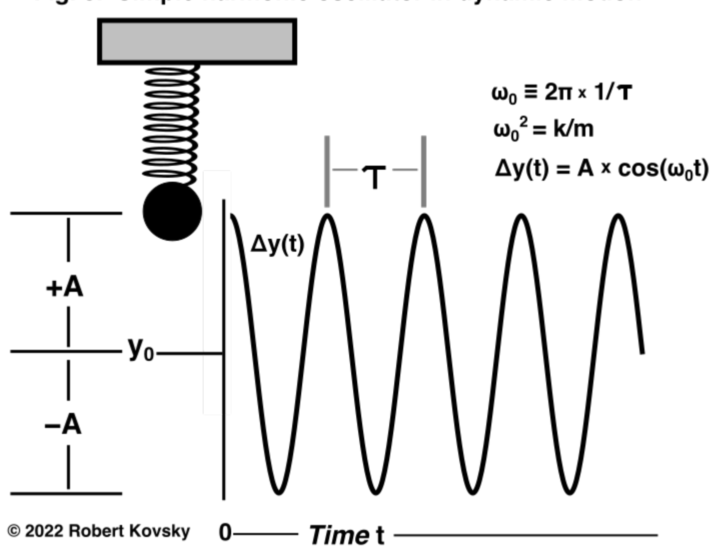
A spring in the unstressed static position or in any stressed static position is in a condition called ***equilibrium***. A static equilibrium condition can be maintained for an indefinite period of time. Movement requires a change in influence. e.g., in **E**.

If investigations are repeated with electric fields of equal strength but directed downward, everything in the new cases is "the same" as formerly, except for some changes in signs: terms that are "+" turn into the same value but "–"; and vice-versa. Originally, the spring was ***pushing***; in the new cases, the spring is ***pulling***. Each movement occurring in the half-range of spring compression has an exact match in the half-range of spring expansion, except for signs. For the ideal spring, the relationship between $\Delta y$ and $F_k$ is "the same" for spring expansion as for spring compression. The position $y_0$ is at the center of the whole range of motion. ["The same" denotes ***identity*** in imaginary domains; a sufficiently close resemblance is often realized approximately and/or temporarily in actual events. An ideal mental spring has "the same" stiffness forever but actual springs eventually fall short of the ideal.]

Next, the paradigm is extended to dynamic movements, where the ball is moving. For the ideal spring, the relationship between $\Delta y$ and $F_k$ is the same for dynamic movements as for static positions.

Suppose that the Fig. 2 system is held in a static position where $\Delta y = +A$, as shown in Fig. 3. Then, suddenly, at $t = 0$, the electric field is removed. The compressed spring pushes the ball down, giving it speed and momentum. As the ball passes through $y_0$, pushing from the spring changes into pulling; and the movement slows until it ***pauses*** at the point of maximum expansion where $\Delta y = -A$.

**Fig. 3. Simple harmonic oscillator in dynamic motion**




After pausing, the ball begins the return trip. On the return trip, the ball speeds up, pulled by the spring, until it passes through the central point $y_0$. Then, pulling changes back into pushing, slowing the ball until it reaches the point of maximum compression of the spring at $\Delta y = +A$, where it pauses before beginning the next cycle of movement. Points of maximum expansion and compression are the same distance A from $y_0$. Next, the whole prior movement is repeated. Because there is no friction, the movement repeats until stopped.

The movement of the ball is summarized in Fig. 3. The governing equation is $F_B = - k \times \Delta y$ or $m \times (d^2\Delta y/dt^2) = - k \times \Delta y$. The solution of this differential equation is shown on the graph with time t on the x-axis and position y(t) on the y-axis. Time starts from 0 when the field is removed. The position of the ball is described by a periodic or cyclical function with a period $\tau$. For a periodic function, $f(t+\tau) = f(t)$, where f(t) is expressly defined over the first period, where $t \in [0,\tau)$. Periods link up in the fashion of repetitive tiles that continue indefinitely.

The ***maximum amplitude*** of the movement (A) is independent of the ***period*** of movement ($\tau$). Movements with different maximum amplitudes have the same $\tau$.

The ***natural frequency of the system*** $\boldsymbol{\omega_0}$ is defined as $2\pi/\tau$. For the SHO, $\omega_0^2 = k/m$. The movement is described by the function $\Delta y(t) = A \times \cos(\omega_0 t)$.

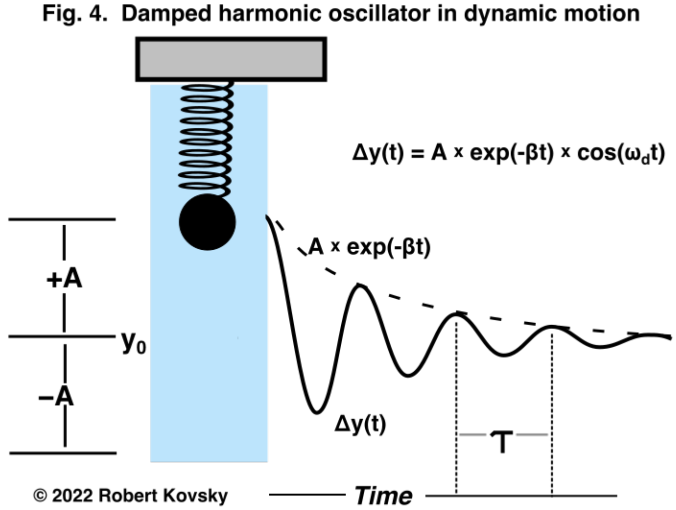
Moments of pause are important features in designs below. During a pause, the ball is not moving. Suppose that, as the ball pauses, a magnetic apparatus enables a researcher to ***stick*** the ball to a background sheet on which the system is mounted. Then, some time later, it is possible to release the ball. When it is released, the ball will resume movement as if no interruption had taken place. The resumed movement is the same as an uninterrupted movement, except for a shift in time. It is only when the ball pauses that it can be stuck and later released for a resumption of identical movement. In final designs of this project, "pausal moments" and "sticking states" become essential features of locomotion movements.

## § 2. Harmonic oscillator with damping and pumping

Dissipation or loss of energy is introduced, arising from friction or viscosity of a surrounding fluid shown in blue in Fig. 4.

Dissipation is based on movement of the ball through the fluid and is denoted by a ***damping parameter*** $\beta$. $\beta$ has a dimension of 1/sec, same as $\omega_0$, and $\beta$ can be compared to $\omega_0$. In Fig. 4, a case of low dissipation, $\beta$ is much less than $\omega_0$.

**Fig. 4. Damped harmonic oscillator in dynamic motion**




The governing equation becomes $m \times (d^2\Delta y/dt^2) = F_B = - (k \times \Delta y) - 2m \times \beta \times (d\Delta y/dt)$. As the ball rises above $y_0$, both the spring and viscosity slow down the movement. For low dissipation, as in Fig. 4, the movement has a succession of high points. The maximum amplitude of $\Delta y$ at successive high points goes down as time proceeds. The period between successive high points, $\tau$, is fixed, thus extending the concept of a periodic function. In symbols, the movement is described by: $\Delta y = A \times e^{-\beta t} \times \cos(\omega_d t)$, where $\omega_d$ is the ***damped frequency***. For low dissipation, $\omega_d$ is slightly less than $\omega_0$. $\omega_d$ approaches $\omega_0$ as $\beta = 0$.

Next, energy is pumped into the damped system to overcome the decline in amplitude. As a useful metaphor, a parent pumps energy into the body of their child riding on a playground swing. Suppose that the movement of the child's body is exactly repetitive, receiving the same push and reaching the same height during each cycle. Once each cycle, the parent provides the needed push. In such metaphor, the applicable paradigm is the ***pendulum paradigm***, which is similar to the SHO paradigm. Like the damped SHO, the swing has friction and the child would come to a resting position if the parent did not pump energy into the system.

For an individual harmonic oscillator in damped dynamic motion, pumping can be provided by an electric field that pulls in the same direction as the spring. Suppose that an upward-directed field is turned "on" just after the moment of pause at maximum expansion of the spring. The field is held "on" until the ball passes $y_0$, when the field is turned "off." The field is thus held "off" for about ¾ of the cycle.

If the "on" strength of the field is held steady for successive cycles, the point of maximum compression of the spring will stay the same. Starting from such repetitive cycles, the point of maximum spring compression can be raised or lowered by increasing or decreasing the strength of the electric field, within a range of movement. Thus, the amplitude of movement is variable, set by a researcher or device operations — while the frequency of movement is tethered to the "natural" or "bodily" timing of movement of the oscillator, which depends chiefly on k, m and $\beta$, with only a minor dependence on the strength of the field.

§ 3. VE movers drive elemental movements of an oscillator module.

a. VE movers provide pumping forces

b. static reference positions of a hybrid force device

c. movement module

d. repertoires of movement of the movement module

e. details of movement production and control

a. VE movers provide pumping forces

In large part, designs in this project do not require a specific source or form for the pumping force. As with motor vehicles, much the same movements can be produced using a variety of energy conversion methods — internal combustion or electricity or steam. In this project, pumping forces are provided by ***VE movers***. Operations of VE movers are congruent with those of VE bursters and modules, subjects of anticipated future investigations as discussed below.

VE mover operations were defined and applied in the wriggler project. For purposes here, a mover is powered by its own source of VE and is driven by a ***bursting device*** or "burster" in a device module. A burster sends a ***pulse burst*** to the mover. Each burst contains n ***pulses***, where the ***pulse number*** n can vary from 1 to 15. A pulse is an instantaneous transfer of VE, like an electrical impulse. A pulse carries one unit of VE and resembles an action potential in nerves.

In response to a pulse burst, a mover produces a ***twitch*** with a force strength that depends on n and on the variable length of the mover. For purposes here, with a specification revised from that in the wriggler project, a burst results in a mover twitch that lasts a specific period denoted $\Lambda$. Definition of $\Lambda$ may depend on the application, n, etc.

While it is being produced, the twitch has a force strength $F = nF_1 - (j \times \Delta L)$, where n is the pulse number and $\Delta L$ denotes the momentary shortening of the mover from its maximum length L. F may change during a twitch. Other mover specifications are $F_1$, which denotes a fundamental unit of force; and j, which denotes a ***dissipation factor***. The dissipation factor j resembles a spring stiffness k but the mover does not conserve energy and has no "potential energy." The burster sends a continuing stream of bursts to maintain a force. If no signals are sent, the mover is ***flaccid*** and has no force production or effect.

For purposes here, $\Lambda$ starts just after the spring starts to compress following a moment of pause and reversal — and ends just after the ball passes $y_0$ — thus lasting for approximately ¼ of a cycle. The mover pulls while the spring is pulling and the mover force augments the spring force for that period.

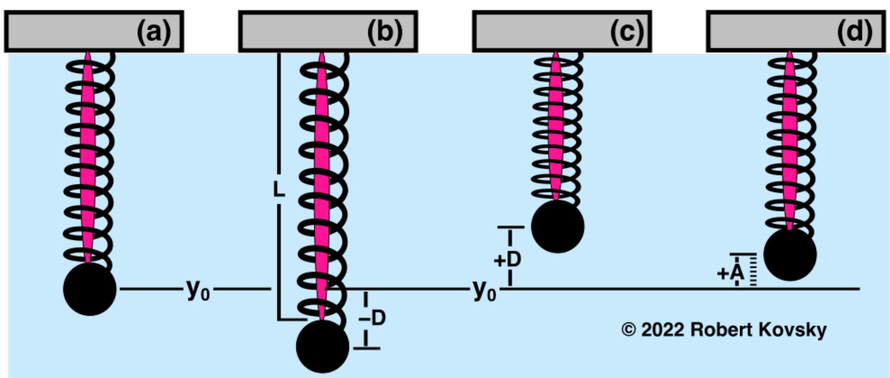
As in other SHO paradigms, the amplitude of a cycle is highly variable (with levels 1 through 15) while the frequency of a cycle is tethered to that of an undriven version and based on k, m and $\beta$, with variations based on other quantities.

b. static reference positions of a hybrid force device

Fig. 5 shows reference positions of a ***hybrid force device*** that combines a Hooke's Law spring with a VE mover. VE movers provide pumping forces for oscillators by twitching during the first half of the compression period of the spring. In Fig. 5(a). 5(b) and 5(c), movers are flaccid and balls are stuck in static positions.

As shown in Fig. 5(a), the static unstressed position of the spring with a flaccid mover is still denoted $y_0$. In Fig. 5(b), the maximum length of the mover is L, which corresponds to an expansion of the spring where $\Delta y = -D$. This defines a strict operational limit of the device; movement in the downward direction is prohibited past this limit. There is a mirror point in the compression half of the cycle where $\Delta y = +D$ (Fig. 5(c)) but no limit applies. The ball can rise above that point if enough energy is pumped in during the ¼ cycle where the spring force is augmented by an active mover, so long as dissipation is sufficiently high that the ball remains above $\Delta y = -D$ during the expansion stroke.

Fig. 5. Reference positions of damped harmonic oscillator augmented with a Virtual Energy mover




In an operating version shown in Fig. 5(d), the point of maximum compression of the spring with n=15 is denoted by "+A" ; +A is less than +D. Fig. 5(d) includes a scale or series of positions from $y=y_0$ to $y=+A$. Each position corresponds to a point of repetitive maximum spring compression produced by twitches driven by pulse bursts with a specific pulse number n.

If pulse number n = 0, there are no pulse bursts; the mover is always flaccid and the ball never moves away from $y_0$. If n is changed from 0 to another number, after a helpful nudge and enough time, the new repetitive position will correspond to the new n. (See discussion on details below.) Whenever the pulse number holds at a steady n > 0, the repetitive maximum position of the oscillator will hold steady. Let $\Delta y_n$ denote the repetitive maximum position corresponding to bursts with pulse number n. There is an orderly progression: $\Delta y_{n+1} > \Delta y_n$.

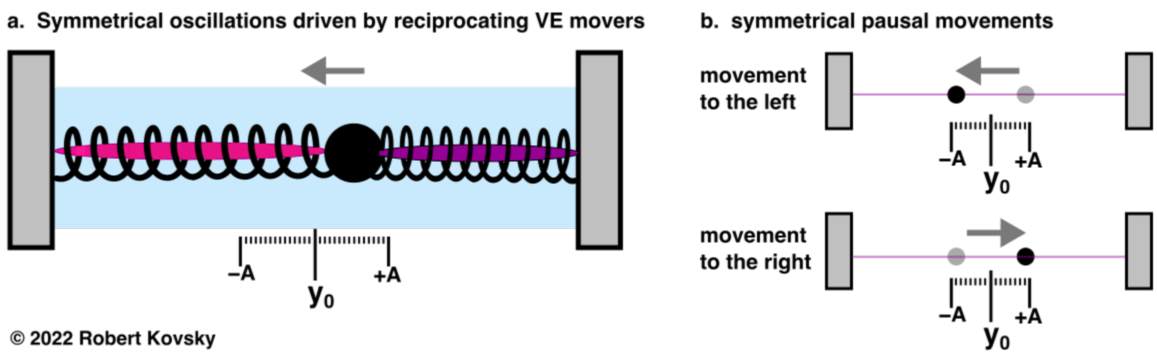
c. movement module and pausal movements

The ***movement module*** shown in Fig. 6(a) becomes a unit of construction in arrays of oscillators in part B. Likewise, ***pausal movements*** shown in Fig. 6(b) become units of construction in wavy locomotion movements produced by such arrays.

The movement module incorporates two identical hybrid force devices in a fixed arrangement that produces multiple repertoires of movement, e.g., symmetrical oscillations. Fig. 6(a) shows the module in a stroke to the left, just after movement has reversed direction and the left mover has started producing a twitch. The twitching mover is bright magenta and the flaccid mover is dull purple.

Movement of the ball pauses at a moment of reversal, which is also given other names depending on the application, "pausal moment," "sticking moment" or "switching moment." At a pausal moment, further action can first be postponed by temporary sticking, and then resumed, the same as if uninterrupted, when the sticking is removed. The length of such a pausal moment ranges from 0 to ∞.

**Fig. 6. movement module and elemental pausal movements**




d. repertoires of movement in the movement module

(i) ***Symmetrical oscillation movements*** are continuous, repetitive back-and-forth movements of the ball. The same number of pulses is sent to left and right movers in alternating streams. Pulse streams, movements and positions of the ball are all symmetrical with respect to $y_0$. There are 15 symmetrical oscillation movements driven by bursts with pulse numbers 1 through 15.

(ii) ***Pausal movements*** are based on oscillatory movements, with the addition of a "sticking feature" that operates at pausal moments. Here, the "sticking feature" is an arbitrary pedagogical invention. A "sticker device" is introduced below that applies similar sticking features in operational designs.

A pausal movement starts with a pausal moment on one side of the module and concludes with a pausal moment on the other side. Symmetrical pausal movements are an important special case.

Symmetrical oscillation movement can be analyzed into alternating symmetrical pausal movements with zero-length pausal moments between pausal movements.

As an initial guess, the period of the movement should be close to $\omega_0$ and tethered to k, m and $\beta$.

Suppose that as symmetrical oscillating movement is ongoing, as shown in Fig. 6a, and that a $\Lambda$ period starts in the left mover just after the ball pauses at position $y_n$. Force produced by the left mover, combined with force from the spring, is just sufficient to overcome dissipation and to carry the ball to position $-y_n$, where again it pauses and again the direction of movement reverses. Then the right mover produces a twitch, the same strength as the prior twitch of the left mover with appropriate changes of sign. The next reversal occurs when the ball pauses again at position $y_n$.

With a sticking feature, a pausal moment can be maintained for an arbitrary or indefinite period of time. A pausal movement begins at the conclusion of one pausal moment and ends at the commencement of the next pausal moment. In continuous operations, each pausal moment follows a pausal movement; and each pausal movement follows a pausal moment. The whole movement has a structure of alternating pausal moments and pausal movements.

The strength of a twitch and the length of a pausal moment are both variable. Both variations are under the arbitrary control of a researcher or other device operations. A change in one variation does not affect the other; they are ***independent***.

(iii) The repertoire of movements is enlarged to include ***changing movements*** that occur after a change in the pulse number from n to m. After a period of change, the new $y_m$ position has been established. See subsection (e) below for more details.

(iv) ***Impulsive movements*** are additional movements caused by pulse burst signals from a researcher or other device operations. These can be arbitrary in form and timing; the class can be defined expansively so as to include all possible movements of the module with operations restricted by $\Lambda$, etc.

One use of impulsive movements is to start movement in a module that is resting at $y_0$. After movement is started, inherent operations of the module take over. Sensory signals discussed in the metamorphosis project can trigger or control impulsive movements that strengthen a weak symmetrical oscillation movement or that speed up completion of a changing movement.

e. details of movement production and control

The hybrid module in Fig. 6 operates according to principles of classical physics, plus augmentation by VE principles. Equations of motion can be solved by standard methods. VE conversions are separable from conserved energy conversions; only conserved energy conversions are used in this project.

From the VE perspective, energy comes in different forms; and devices convert energy between forms. No new energy appears in any conversion; rather, some energy is lost or dissipated in every actual conversion. In idealized paradigms, however, energy loss during a conversion may be presumed to be zero. This presumption helps to clarify paradigms and constructions. Presumptively, practical workarounds make actual applications possible despite actual energy losses.

In this project, energy comes in the following forms:

1. Potential energy stored in springs;
2. Kinetic energy carried by masses in motion;
3. Energy carried away by dissipation based in a viscous fluid;
4. Work performed by movers; and
5. Virtual Energy for movers from arbitrary sources.

Energy conversions of concern here are:

a. Ideal springs inter-convert potential energy of a spring and kinetic energy of a moving mass, based on the force relation: $m\times(d^2\Delta y/dt^2) = F_B = -(k\times\Delta y)$.

b. A viscous fluid converts kinetic energy of movement into dissipated energy, reducing the force on the mass by $\Delta F_a = -2m\times\beta\times(d\Delta y/dt)$ .

c. Movers convert VE into kinetic energy based on a force relation that adds to the force on the mass by $\Delta F_b = nF_1 - (j\times\Delta L)$ for the period $\Lambda$ in a cycle.

During a pausal moment, the only energy is potential energy in springs. The class of symmetrical pausal movements has 15 values for such maximum potential energy, corresponding to 15 sizes of pulse bursts. Potential energy is the same at the end of such a movement as at the beginning. Summed over a symmetrical pausal movement, and applying a conservation principle, the kinetic energy provided by a working VE mover equals the kinetic energy lost through dissipation.

Analysis of changing movements illustrates details of operations. Suppose that the module is driven by bursts with n=3 and is producing symmetrical oscillatory movements of size $y_n = \pm 3$. Suddenly, n changes from n=3 to n=12, which is thereafter maintained in a steady stream. Perhaps the first n=12 twitch starts when the mass is at $y_n = +3$. A spring with low n holds a relatively small amount of potential energy. The extra mover force carries the mass beyond $y_n = –3$ but the small contribution from potential energy leaves it short of $y_n = –12$. The next twitch will carry it closer to $y_n = + 12$, but perhaps still short. Eventually, $y_n = +12$ will be established, perhaps with the assistance of an impulsive movement.

Conversely, suppose that the maximum position of the mass is $y_n = \pm 12$, maintained by steady n=12 pulse bursts. Suddenly, n changes to n=3. With dissipation occurring during a large travel path, the resulting mover force is insufficient to carry the ball to $y_n = –12$, although still beyond $y_n = –3$. In successive cycles, movement dwindles down to $y_n = \pm 3$. An impulsive twitch by an opposing mover will brake the movement if faster dwindling is desired.

Similar courses of movement occur as to any asymmetrical movement that appears in the symmetrical module driven by symmetrical signals: symmetrical forces and operations extinguish asymmetries.

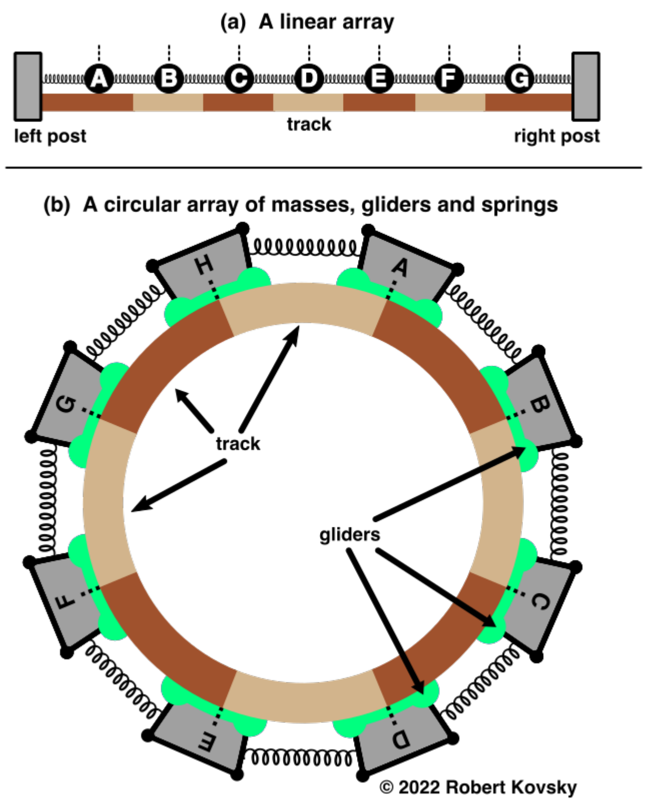
4. Compression waves in arrays of alternating springs and masses

Simple arrays are made of alternating masses and springs. Fig. 7(a) shows a linear array where masses slide on a frictionless ***track*** and where the masses on the ends (A and G) are secured to immobile ***posts***. Fig. 7(b) shows a circular array where the masses ride on ***gliders*** on a frictionless track.

Masses in an array are all the same except for letters that name them. Springs are all the same. Both arrays produce ***compression waves***, where springs expand and compress without bending and masses push and pull each other along direct lines. Movements are constrained to occur in one dimension, whether linear or circular.

Dotted black lines indicate positions. Masses and springs in Fig. 7 arrays are in central unstressed static positions.

**Fig. 7. Arrays of alternating masses and springs**

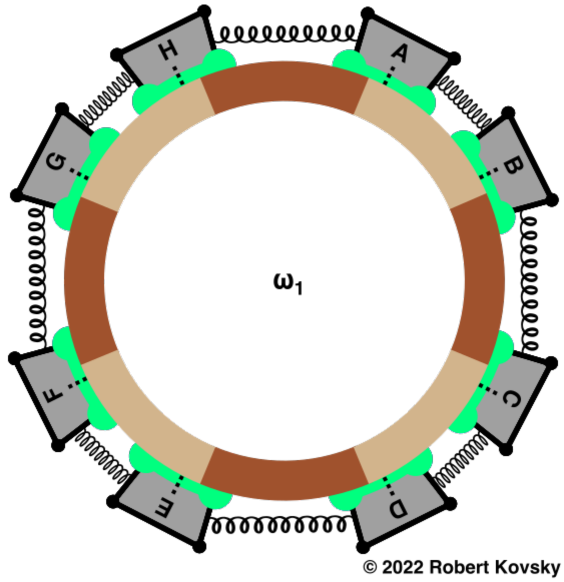



Mathematical investigations of the two arrays are similar, showing that movements can be organized by means of ***normal modes***. A normal mode is a simple kind of movement characterized by a single frequency. Movements other than normal modes are constructed mathematically from normal modes and combine multiple normal modes and multiple frequencies.

Further development proceeds with the circular array. Discussion returns to the linear array for concluding designs.

Fig. 8 shows a normal mode of oscillation in the 8 unit circular array. For purposes of notation, let Spr(A,B) denote the spring between mass A and mass B and so forth for other springs. In this mode of oscillation, movements of 4 springs replicate each other [Spr(A,B), Spr(C,D), Sp(E,F), Spr(G,H)]: and movements of the other 4 springs replicate each other [Spr(B,C), Spr(D,E), Spr(F,G), Spr(H,A)]. The two sets of springs have exactly opposite operations. While Spr(A,B) is compressing, Spr(H,A) and Spr(B,C) are expanding.

**Fig. 8. A normal mode of oscillation in a circular array**



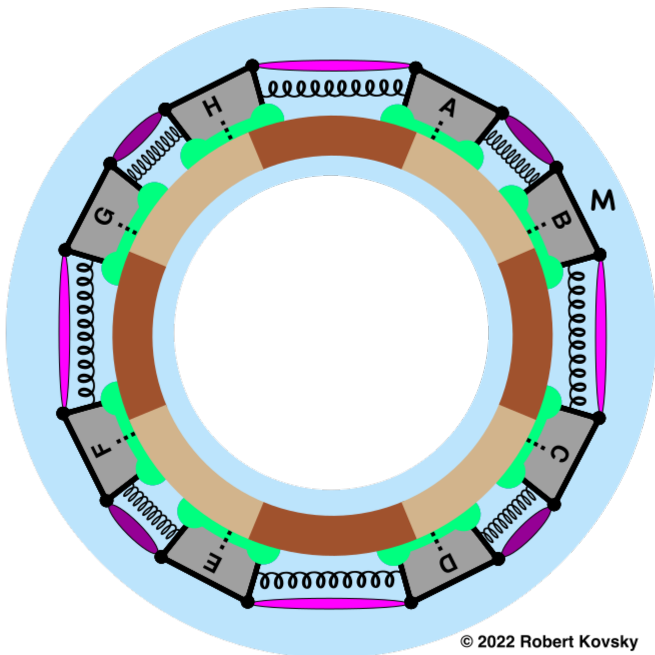

In the normal mode represented in Fig. 8, all movements repeat with a single natural frequency $\omega_1$. Fig. 8 shows positions at a particular instant, like a snapshot that is taken just before a pause. In the next few instants, springs that have been expanding — will pause — and then begin to compress; springs that have been compressing — will pause — and then begin to expand.

5. VE movers drive compression waves in the circular array.

The next stage of the circular array construction parallels the construction of the individual SHO above, adding a dissipative medium and VE movers that drive oscillations, as shown in Fig. 9. All 8 movers are the same kind of device. Movers are named in the style of springs, e.g., mover (A,B).

The instant shown in Fig. 9 is just after movers have been activated, which occurs just after a moment of reversal.

**Fig. 9. VE movers driving oscillations in a damped circular array**




The dissipative medium damps and extinguishes all movements except oscillatory movements caused by movers. Mover forces are applied for about ¼ of a cycle, starting from just after a pausal moment of maximum expansion and ending when the length of the mover becomes less than $y_0$. Movers are flaccid during the rest of the cycle. In Fig. 9, active movers are bright and flaccid movers are dull.

Mover forces are initially small compared to the spring force and are initially applied at a frequency equal to the natural frequency $\omega_1$ as measured in a Fig. 8 version that has neither damping nor pumping.

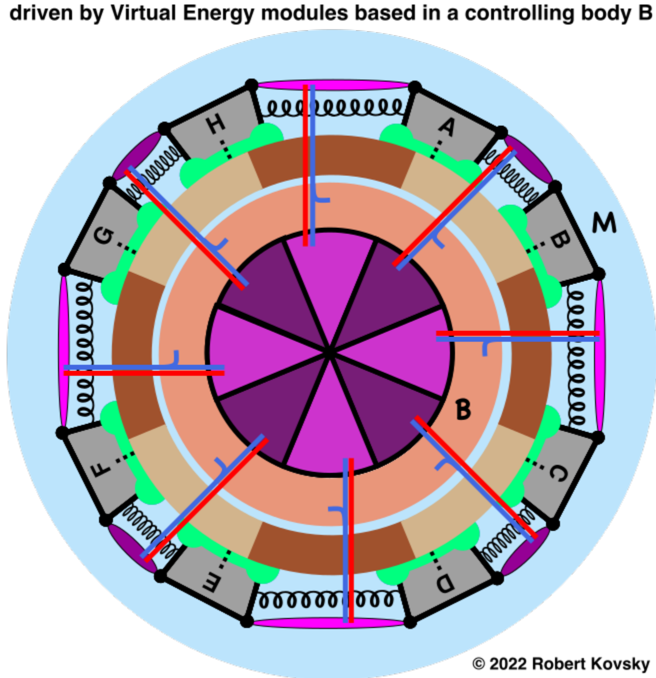
Then, in processes of bodily experimentation, the frequency is varied bit by bit and changes in maximum amplitudes of movements are measured by means of sensory devices discussed in the metamorphosis project. According to the physics paradigms, if dissipation is low, the resonant frequency that maximizes the amplitude of movements should be found close to the natural frequency $\omega_1$.

6. A controlling body of VE modules drives movers and compression waves.

The construction next adds VE modules that drive the movers. In Fig. 10, collective operations are based in the central body B that also contains 8 individual modules in the form of slices of a pie-shaped group. Red lines denote projections that carry drive signals from a module to a mover. Blue lines denote projections that carry sensory signals from a mover to a module and to the body B.

Operations of modules are influenced both by individual sensor signals and also by collective conditions of B. Collective conditions of the body are subject to changes that can be based in the environment, internal operations or a researcher. As a result of changes in collective conditions, movements are changed and repertoires of movements are changed. Future investigations aim to develop methods for combining individual and collective influences in modular operations.

**Fig. 10. Oscillations in a circular array of masses and springs driven by Virtual Energy modules based in a controlling body B**




In Fig. 10, alternating discharges in modules drive alternating force patterns in movers. Bright modules discharge pulses into active movers during early compression of a spring. Dull modules are silent; their movers are flaccid.

The pattern of discharges in the group of modules in Fig. 10 resembles a normal mode of vibration of a drum head or a membrane in musical acoustics. Anticipated investigations involve operations of a group of modules with individual and collective aspects that depend on attributed material properties of B. Adapting standard paradigms, degrees of freedom include mode of movement, frequency of stepping and direction of movement (clockwise or counter-clockwise).

7. Wavy locomotion movements in circular and linear arrays are produced by hybrid designs of classical parts and VE parts

a. Glicker devices and sticking states

b. Repertoires of movement produced by a circular array with glickers

c. Application to a linear array of masses, springs, VE movers and glickers

d. Whole-body wavy locomotion movements of the linear array

e. Ripple-wave locomotion movements of the linear array

f. Anticipations of control bodies and entrained sensory-mover modules

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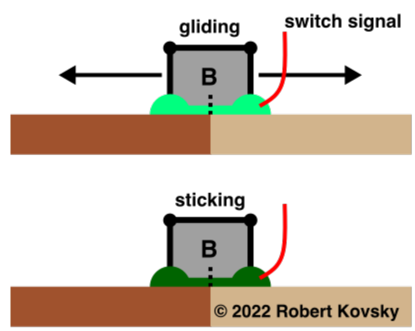
a. Glicker devices and sticking states

Fig. 11 shows a new device, the ***glicker device*** or "glicker," replacing the glider device used previously. The glicker adds a new ***sticking state*** or "sticker" to a glider. A sticker is fixed to the track; lateral forces cannot move it at all.

The glicker device switches between two states:
(1) the bright green "glider" state that is mobile; and
(2) the dull green "sticker" state that is immobile.

A red projection carries a ***switch signal*** from a VE control module. A switch signal activates a toggle: a gliding state changes into a sticking state; or a sticking state changes into a gliding state.

Fig. 11. A glicker device switches between a gliding state and a sticking state




A restriction on operations is that the state of a glicker device can change only at a moment when the device is stationary or when movement pauses, e.g., during a pausal moment as springs change from expanding to compressing or vice-versa. In other words, a switching moment can accompany a pausal moment. Movers must be flaccid during a switching moment.

Perhaps the timing of the switching moment is controlled by a researcher. Perhaps a module can be trained to switch according to a sensory cue. Perhaps a glicker device can hold a switch pulse and then switch states when a pause occurs.

[In similar movements of a person walking slowly with small steps, modeled by the pendulum paradigm, the forward foot pauses just above the ground at the end of a stride; then, it is brought down and fixed to a foothold.]

b. repertoires of movement produced by the circular array with glickers

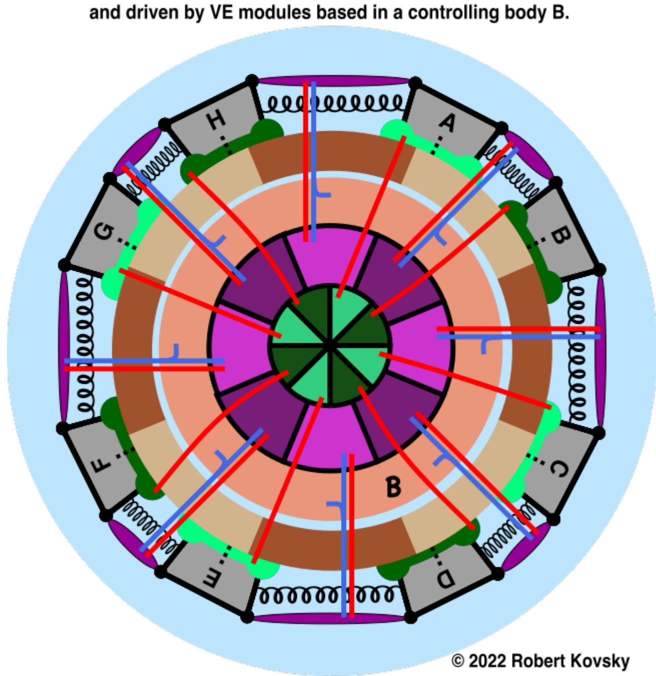
Fig. 12 shows the final design in the line of development using the circular array. Gliders have been changed to glickers. In the center of B, a new pie-shaped group of green modules controls the glickers. This design produces multiple repertoires of movement. Fig. 12 shows a moment when all movers are flaccid.

1. If all glickers are maintained as gliders, the new design produces the same oscillatory movements as the Fig. 10 design.

2. Movement repertoires of the individual mover module (Fig. 6 in § 3) reappear in an array of mover modules when certain glickers stay in the sticking state for extended periods called ***fixed sticker states***. Fixed sticker states divide the circular array into independently operating subsets.

In Fig. 12, suppose that glickers B, D, F and H are kept in the sticking state. Movement of mass A between H and B is the same as that of the ball in the movement module in Fig. 6a, mapping a line into an arc of a circle. The same movement occurs with masses C, E and G. Movements of A, C, E and G can be independent of each other or they can be driven collectively.

**Fig. 12. Wavy locomotion movements are produced by a circular array of masses, springs and VE movers riding on glicker devices and driven by VE modules based in a controlling body B.**



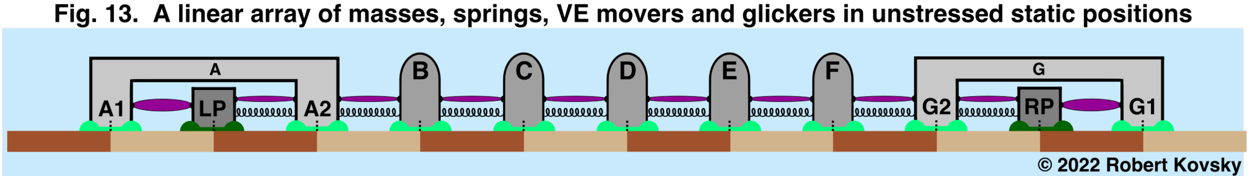

Various repertoires of movements are produced when different sets of glickers stay in fixed sticker states, e.g., when glickers D and H are kept in fixed states and all other masses ride on gliders.

3. Locomotion movements. Let Fig. 12 represent a moment when masses A, C, E and G are moving clockwise and approaching a pausal moment. In the moment just after that shown in the figure, let glickers A, C, E and G switch into sticking states so that the system pauses. Then glickers B, D, F and H switch into gliding states and start moving with VE assistance — until they pause at the end of the step and switch to sticking. Successive pausal moments and pausal movements produce a clockwise locomotion movement. Locomotion movements can be reversed and similar counter-clockwise movements are produced.

c. Application to a linear array of masses, springs, VE movers and glickers

The linear array shown in Fig. 13 produces repertoires of movements that correspond to those produced by the circular array and provides clearer examples of locomotion movements.

**Fig. 13. A linear array of masses, springs, VE movers and glickers in unstressed static positions**




Looking at arrays holistically, the Fig. 13 array is identical to that shown in Fig. 7a. The short masses "LP" (Left Post) and "RP" (Right Post) serve the same function in Fig. 13 as "posts" in Fig. 7a; their state of immobility is indicated by the dull green sticker state of the glickers that hold them. The size of the LP/RP mass is chosen for easy production of "sliding movements" discussed below.

In Fig. 13, all eight springs have the same k. All eight internal movers are the same (thus excepting the two larger movers at the ends). All masses A through G have the same m value. Masses A and G are spread out in larger-sized bodies and each rides on two gliders but these differences do not affect movements.

d. Whole-body wavy locomotion movements of the linear array

The Fig. 13 linear array produces desired locomotion movements. First, operations are shown for central segments shown in Fig. 14. Operations at the ends of the linear array are then defined to match movements of the center.

**Fig. 14. internal segments of a linear array in unstressed static positions**

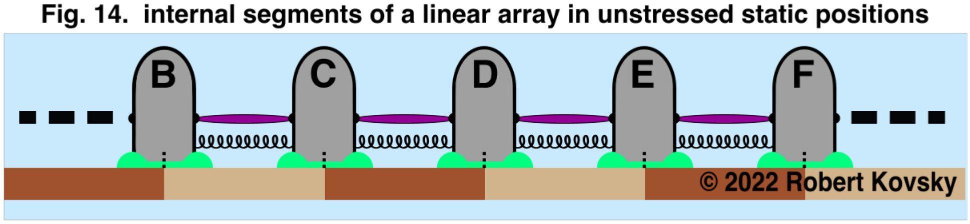
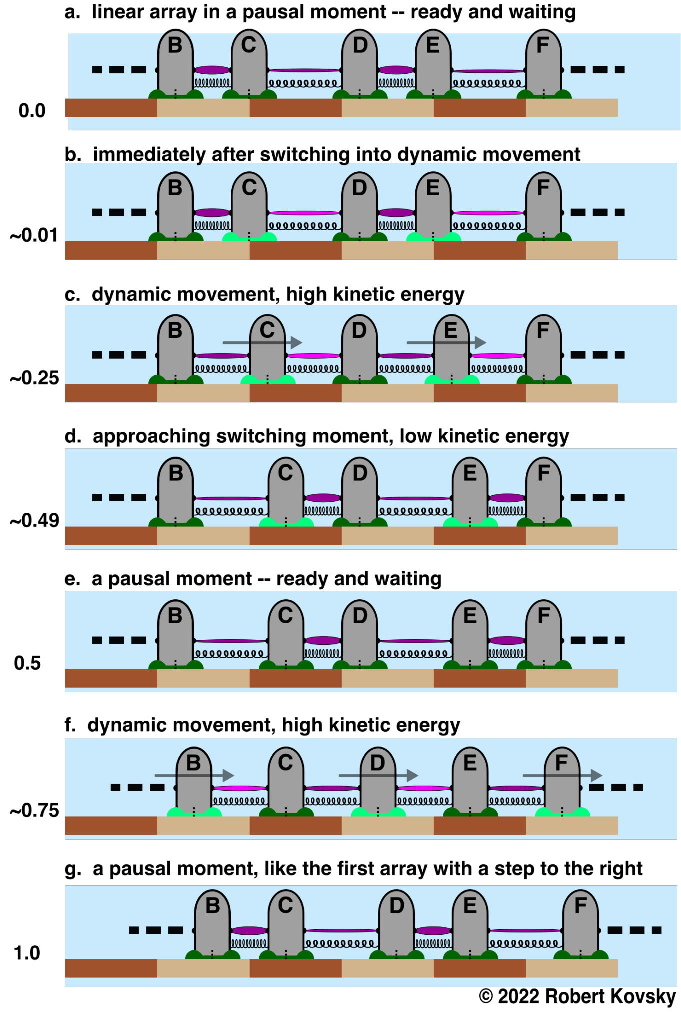


Fig. 15 shows a stepping cycle in a locomotion movement of the central segments.

In Fig 15(a), the cycle starts with a pausal moment of indefinite length. All glickers are in the sticking state. Potential energy is stored in springs. Movers are flaccid. The cycle clock on the left reads 0.0. A cycle will be completed when the cycle clock reads 1.0.

Fig. 15. a step in a locomotion movement of a linear array




(b) In the first moment of the cycle, glickers C and E switch to the gliding state; next, movers (C,D) and (E,F) are activated.

(c) Dynamic movement is occurring. Springs are close to $y_0$. After movers (C,D) and (E,F) switch off in the next moment, all energy will be kinetic. The clock reads about about 0.25.

(d) Clock approaches 0.5 and a pause. Kinetic energy is low and springs are charged with potential energy. Movers are flaccid.

(e) Gliders C and E change to stickers; a pausal moment follows, similar to conditions in (a).

(f) The second dynamic movement in the cycle. Like conditions in (c), springs are close to $y_0$. After movers switch off, all energy will be kinetic.

(g) Gliders B, D and F switch to stickers and all segments enter into another pausal moment.

The size of a step is a variable that depends on the pulse number n that drives all the movers. If glickers A, C, E and G are kept in fixed sticker states, a change in pulse numbers will charge or dissipate energy in springs and change the size of steps. (See discussion in § 3.) The amount of time required for a step varies little with n and depends chiefly on k, m and β. Fastest movements occur when pausal moments have zero length.

Fig. 16 and Fig. 17 show movements of end pieces that occur synchronously with those of Fig. 15, omitting stages b. and d that are shown in in Fig. 15.

In Fig. 16(c), masses E and G are in dynamic movement.  G's second glider and the distribution of mass in G  do not affect the movement.  The mover and spring between G2 and RP operate the same as those between E and F.

The movement of RP in Fig. 15(f) is a new ***sliding movement*** that occurs inside G, which is fixed.  The force produced by the big mover between RP and G1 is augmented by the spring and opposed by the other mover.  Movers and springs in such a configuration produce a spectrum of balancing positions that are subject to multiple possible adjustments.

**Fig. 16.   a step of the front parts of the linear array**

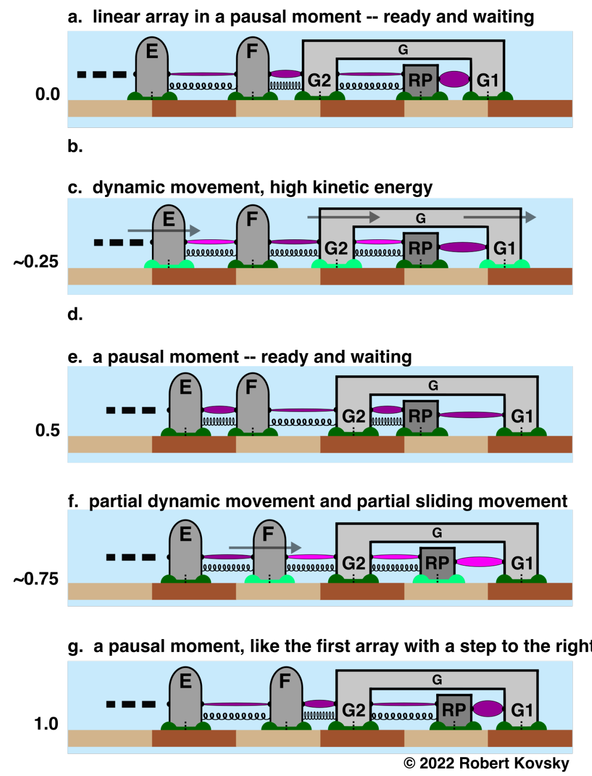
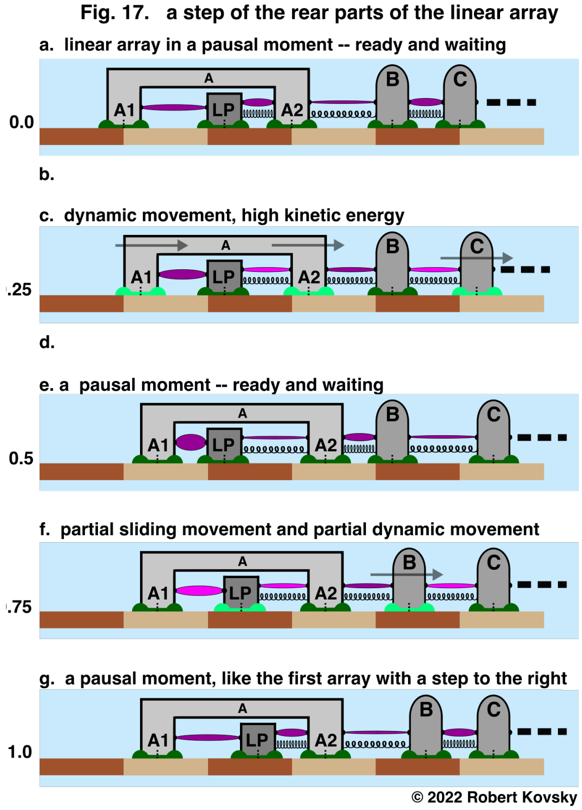



**Fig. 17.   a step of the rear parts of the linear array**



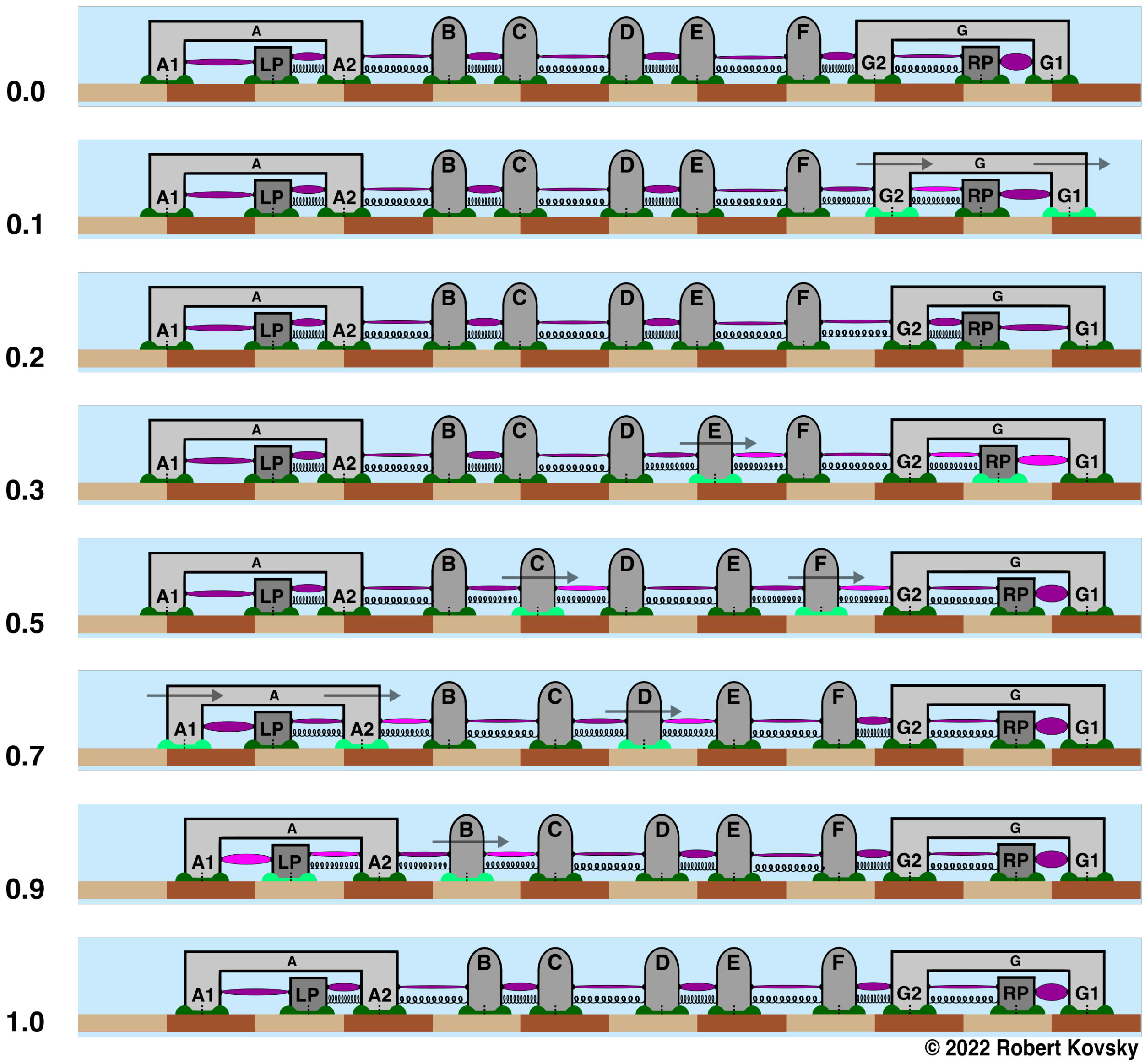

A sliding movement is a quasi-static movement rather than a dynamic movement.  The sole function of movers (G1, RP) and (A1, LP) is to perform sliding movements and sufficient strength is presumably provided.  Operations do present questions of timing.  A and G glickers are fixed for ¾ of a cycle and this should be sufficient for slower controlled movements of the posts.

e. Ripple-wave locomotion movements in the linear array

Another mode of locomotion movements is called ***ripple-wave locomotion***, where a local locomotion movement passes through the array in sequenced sets of movers, first through front parts and then through rear parts. (Ripple-wave movements can also start at rear parts and pass towards the front. Movements discussed in this project can generally be produced in the direction contrary to that shown.)

In the cycle shown in Fig. 18, pausal moments occur when the cycle clock is at 0.0, 0.2, 0.4, 0.6, 0.8 and 1.0. The figure mostly shows central dynamic moments.

**Fig. 18. A step in a ripple-wave movement in the linear array**



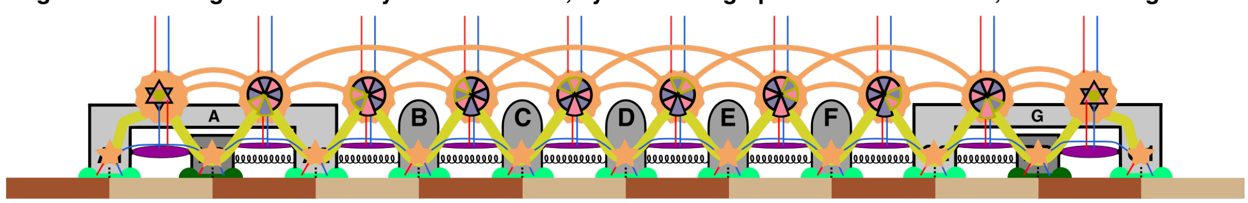
f. Anticipations of control bodies and entrained sensory-mover modules

This project is chiefly concerned with adaptations of standard physics paradigms. The only VE devices are movers that produce forces conformable to conserved energy principles. A secondary concern is preparation for anticipated development of VE burster modules and controlling bodies, as suggested in Fig. 10 and Fig. 12.

Fig. 19 shows possible developments for a control system in the linear array in a general conceptual design. Each internal mover is driven by a module in the shape of a sector of a circle, resembling a piece of a pie. The driving module is part of a group of modules that makes up the whole pie or circle, resembling the design in Fig. 10. Each group of modules holds a representation of the whole array of 8 internal movers. The driving module is larger and more complex than other modules in the group. A group of modules is based in a common ***control body*** and all modules operate ***synchronously***, that is, with a common rate and coordinated values. Each control body is specific to one mover. Modules sharing a control body manifest both individual operations and controlled collective operations.

A different kind of control body and module sends switching pulses to glickers. End pieces of the array also incorporate special control bodies and mover modules.

**Fig. 19. A full-length control body entrains a beat, synchronizing operations of modules, movers and glickers**






In Fig. 19, all the various control bodies, mover modules and glicker modules are maintained in synchronous operations through a principle of ***entrainment***. Entrainment was first studied by Christiaan Huygens (1629-1695) as to pendulum clocks and the concept was soon extended to spring-driven clocks. Perhaps several identical spring-driven clocks are placed on a single table; soon their ticking adjusts so that they all tick together. Numerous studies suggest that collective vibrations in the table accomplish this entrainment and that physical properties of the table may determine whether entrainment occurs and specific details of entrained operations.

Applying a principle of entrainment, the yellow full-length sinous control body in Fig. 19 maintains a whole-body ***beat*** that produces synchronous discharges of pulses in all control bodies and modules (or in sequenced sets, as in a ripple-wave), which thus participate in both collective operations and individual operations.